Homework #5 (Due on September 25th, 2019)

1. Consider an individual with Cobb-Douglas preferences $u(x_1, x_2) = (x_1 x_2)^{0.5}$, where x_1 and x_2 denote the amounts consumed of goods 1 and 2, respectively. The prices of these goods are $p_1 > 0$ and $p_2 > 0$, respectively, and this individual's wealth is $w > 0$. The government needs to collect a large amount of money to finance a new Health Care plan (any similarity with reality is pure coincidence), and contemplates two options: (1) introduce an income tax equivalent to 40% of individuals' wealth; or (2) charge a sales tax over the price of good 1 (e.g., fuels) which would imply an increase in the price of good 1 from p_1 to $p_1(1+t)$, collecting the same dollar amount as with the income tax. Using the indirect utility function of this individual under option 1 (income tax) and option 2 (sales tax), explain which tax produces a smaller utility reduction.

2. The profit function, $\pi(p)$, is defined as

$$\pi(p) = \max \{p \cdot y \mid y \in Y\}$$

   or alternatively, $\pi(p) \geqslant p \cdot y$ for every feasible production plan $y \in Y$.

   (a) Show that the profit function $\pi(p)$ is convex in prices.

   (b) Prove that Hotelling's lemma holds for this profit function.

   (c) Show that, if the output function $y(p)$ is differentiable at $\overline{p}$, then $D_p y(\overline{p})$ is a symmetric and positive semidefinite matrix.

3. Consider a firm with production function $q = \sqrt{z}$, using one input (e.g., labor) to produce units of output q. The price of every unit of input is $w > 0$, and the price of every unit of output is $p > 0$.

   (a) Set up the firm's profit-maximization problem (PMP), and solve for its unconditional factor demand $z(w, p)$.

   (b) What is the output level that arises from using the amount of inputs $z(w, p)$? Label this output level $q(w)$.

4. Consider a Cobb-Douglas production function $f : \mathbb{R}^2_+ \longrightarrow \mathbb{R}_+$, given by $f(z) = 2^{3/4} z_1^{1/4} z_2^{1/4}$, where $z_1 \geqslant 0$ and $z_2 \geqslant 0$ denote inputs in the production process.

   (a) Check if the production function has nonincreasing, nondecreasing, or constant returns to scale.