Homework #4 (Due on September 18th, 2019)

1. A consumer has a Cobb-Douglas utility function

$$u(x_1, x_2) = x_1^{1/2} x_2^{1/2},$$

where good x_1 represents her consumption of alcoholic beverages, and x_2 is her consumption of all other goods. The price of alcohol is $p > 0$, and the price of all other goods is normalized to 1.

   (a) Set the consumer's expenditure minimization problem. Find first order conditions, and find her optimal consumption of goods x_1 and x_2.

   (b) Substituting your results from part (a) into your objective function, find the expenditure function $e(p_1, p_2, U)$ for this consumer.

   (c) Let us now consider a proposal to reduce the price of alcohol from $p = \$2$ to $p = \$1$ per unit. If the new utility enjoyed by the consumer after the price change is $U = 100$, what is the equivalent variation of this price change?

2. Samantha often consumes two goods during the exam period at the school to relax, that is, chocolate (c) and music (m). Her utility from consuming these two goods is represented by the following quasilinear utility function $u(c, m) = c + 2m^{\frac{1}{3}}$. Her income level during the exam week is $I = \$120$, and the price of a bar of chocolate is $p_c = \$4$. Identify the Compensating and Equivalent Variation when the price for downloading music increases from $p_m = \$2$ to $p'_m = \$3$.

3. Consider a consumer with quasilinear utility $u(x, y) = 2x^{1/3} + y$. The demand for good x from the EMP yields $x^E(p_x, p_y, u) = \left(\frac{2p_y}{3p_x}\right)^{3/2}$, as solved for in Exercise 5.16 (where goods m and c are relabeled as $x = m$ and $y = c$).

   (a) Find demand for good y from the consumer's EMP.

   (b) Calculate the CV for a price increase from $p_x = \$5$ to $p'_x = \$10$, where $u = 30$ and $p_y = \$1$.

   (c) Calculate the CV of the above price change in good x, but using the demand function of good y to see how the consumer's welfare in her purchases of good y is affected by a more expensive good x.

4. Show that the compensating and the equivalent variation coincide when the utility function is quasilinear with respect to the first good (and we fix $p_1 = 1$). [*Hint*: Use the

definitions of the compensating and equivalent variations in terms of the expenditure function (not the hicksian demand). In addition, recall that if $u(x)$ is quasilinear with respect to good 1, then we can express it as

$$u(x) = x_1 + \phi\left(x_{-1}\right),$$

where x_{-1} represents all the remaining goods, $l = 2, 3, ..., L$.]