# Homework #3 (Due on September 11th, 2019)

1. Show that if the demand function $x_i(p, w)$ satisfies the uncompensated law of demand (ULD), then the matrix of derivatives $D_p x_i(p, w)$ is negative semidefinite. [*Hint:* Totally differentiate the Walrasian demand function $x_i(p, w)$ with respect to $p$ and $w$, and use the uncompensated law of demand to cancel some terms. Justify.]

2. Suppose that demand and utility functions are differentiable, and preferences are homothetic. Show that:

   (a) $D_p x_i(p, w_i) = S_i(p, w_i) - \frac{1}{w_i} x_i(p, w_i) \cdot x_i(p, w_i)^T$

   (b) $dp \cdot S_i(p, w_i) \cdot dp = 0$ when $dp = \alpha p$ (when the price change $dp$ is proportional to the initial price level).

   (c) $dp \cdot x_i(p, w_i) > 0$ when $dp = \alpha p$ (when the price change $dp$ is proportional to the initial price level).

3. Consider an individual facing price vector $p = (p_1, p_2) >> 0$ and income $w > 0$. If, after solving his UMP, his indirect utility function is $v(p, w) = \left(p_1^\alpha p_2^{1-\alpha}\right) w$, show that his utility function $u(x)$ must have a Cobb-Douglas representation, where $x = (x_1, x_2)$.

4. Consider a consumer with the following expenditure function

$$e(p, u^0) = g(p) + \left[u^0 \times f(p)\right]$$

   where functions $g(p)$ and $f(p)$ depend on the price vector $p$ alone. Show that a 1% increase in wealth leads to exactly a 1% increase in consumption (i.e., the income elasticity, $\varepsilon_{x_i,w}$) converges to one when the consumer's wealth level tends to infinity, i.e., $\lim_{w \to \infty} \varepsilon_{x_i,w} = 1$.