Homework 1 - EconS 501 (Due Wednesday, August 25th)

1. **[Checking properties of preference relations-I].** In an alternative universe, dolls are anthropomorphized and chosen for a single child, while defective dolls are dropped into the remote town of Uglytown. Among these dolls is the idealistic Molly, who dreams being chosen for a child, despite Uglytown's Mayor Oy assuring her this is a myth. In order to be chosen by a kid they need to perform two tasks: (1) master the song skidamarink (task x) and (2) dress up like the perfect dolls (task y). Molly is very motivated and she prefers a combination of these two tasks that contains more of dress up, i.e., $(x_1, y_1) \succsim (x_2, y_2)$ if and only if $y_1 \geq y_2 + 1$. For this preference relation in the performance of two tasks to become the perfect doll (tasks x and y): describe the upper contour set, the lower contour set, the indifference set of bundle $(2, 3)$, and interpret them. Then check whether this preference relation is rational (by separately examining whether they are complete and transitive), monotone, and convex.

2. **[Checking properties of preference relations-II].** Consider the following preference relation defined in $X = \mathbb{R}^2_+$. A bundle (x_1, x_2) is weakly preferred to another bundle (y_1, y_2), i.e., $(x_1, x_2) \succsim (y_1, y_2)$, if and only if

$$\min \{3x_1 + 2x_2, 2x_1 + 3x_2\} \geq \min \{3y_1 + 2y_2, 2y_1 + 3y_2\}$$

1. (a) For any given bundle (y_1, y_2), draw the upper contour set, the lower contour set, and the indifference set of this preference relation. Interpret.

   (b) Check if this preference relation satisfies: (i) completeness, (ii) transitivity, and (iii) weak convexity.

3. **[Convexity and Strict Convexity].** Explain convexity and strict convexity in preference relations, and compare them. Provide an example where a bundle x is preferred to bundle y when preferences satisfy convexity, but x is not necessarily preferred to y under strict convexity.

4. **[Lexicographic preference relation].** Let us define a lexicographic preference relation in a consumption set $X \times Y$, as follows:

$$(x_1, x_2) \succsim (y_1, y_2) \text{ if and only if } \begin{cases} x_1 > y_1, \text{ or if} \\ x_1 = y_1 \text{ and } x_2 \geqslant y_2 \end{cases} \tag{1}$$

Intuitively, the consumer prefers bundle x to y if the former contains more units of the first good than the latter, i.e., $x_1 > y_1$. However, if both bundles contain the same amounts of good 1, $x_1 = y_1$, the consumer ranks bundle x above y if the former has more units of good 2 than the latter, i.e., $x_2 \geq y_2$. For simplicity, assume that both components have been normalized to $X = [0, 1]$ and $Y = [0, 1]$.

(a) Show that the lexicographic preference relation satisfies rationality (i.e., it is complete and transitive).

(b) Show that the lexicographic preference relation $\succsim$ *cannot* be represented by a utility function $u : X \times Y \to \mathbb{R}$.

(c) Assume now that this preference relation is defined on a *finite* consumption set $X = X_1 \times X_2$, where $X_1 = \{x_{11}, x_{12}, ..., x_{1n}\}$ and $X_2 = \{x_{21}, x_{22}, ..., x_{2m}\}$. [*Hint*: You can define a function $N_i(x_{ij})$ as the number of elements in sequence X_i prior to element x_{ij}; that is,

$$N_i\left(x_{ij}\right) = \#\left\{y \in X_i | y < x_{ij}\right\}.$$

Then define a utility function

$$u(y_1, y_2) = mN_1(y_1) + N_2(y_2), \text{ where } m > 0,$$

and for any pair $(y_1, y_2) \in X_1 \times X_2$.] Show that this utility function represents the lexicographic preference relation.