Homework # 2 EconS501 [Due on September 4th, 2019]

Instructor: Ana Espinola-Arredondo

1. Provide an example where a choice structure, $C(B)$, satisfies WARP but preferences are not rational.

2. Show that a monotonic transformation of $u(x, y)$ does not necessarily satisfy the property of diminishing marginal utility.

3. Consider a consumer with CES utility function

$$u(x_1, x_2) = [x_1^\rho + x_2^\rho]^{\frac{1}{\rho}}$$

where coefficient ρ satisfies $\rho \neq 0$ and $\rho \leq 1$.

   (a) Find the Walrasian demands of this consumer, $x_1(p, w)$ and $x_2(p, w)$.

   (b) What is the Walrasian demand of any good $i = \{1, 2\}$ when parameter $\rho \to 0$?

4. Consider the following demand functions for a consumer facing K goods, and where p_i denotes the price of good i, and p_{-i} represents the vector of prices different from p_i, i.e., $p_{-i} = (p_1, p_2, ..., p_{i-1}, p_{i+1}, ..., p_K)$. State conditions under which these demand functions satisfy Walras' Law.

   (a) $x_i(p_i, p_{-i}, w) = \frac{1}{p_i}\left[\alpha_i + \beta_i w + \sum_{j=1}^{K} \gamma_{ij} p_j\right]$

   (b) $x_i(p_i, p_{-i}, w) = \frac{w}{p_i}\left[\alpha_i + \beta_i \log w + \sum_{i=1}^{K} \gamma_{ij} \log p_j\right]$

   (c) $x_i(p_i, p_{-i}, w) = \frac{1}{p_i}\left[\alpha_i + \beta_i w + \gamma_i w^2\right]$

5. Check whether the following demand functions satisfy the Weak Axiom of Revealed Preference (WARP).

   (a) "Random demand": For any pair of prices p_1 and p_2 and wealth w, the consumer randomizes uniformly over all points in the budget frontier.

   (b) "Average demand": The expected "random demand" given p_1, p_2 and w.

   (c) "Conspicuous demand": The individual spends all his wealth on the most expensive good. This is often referred to as "conspicuous consumption" and includes items such as luxury cars, yachts and private islands! For instance, if good 1 is the most expensive, $p_1 > p_2$, then the consumer spends all his wealth on good 1, $x_1(p_1, p_2, w) = \frac{w}{p_1}$, but nothing on good 2, $x_2(p_1, p_2, w) = 0$. More generally, for any p_1, p_2 and w, the demand for good $i = \{1, 2\}$ is

$$x_i(p_1, p_2, w) = \begin{cases} \frac{w}{p_i} & \text{if } \frac{w}{p_i} = \min\{\frac{w}{p_1}, \frac{w}{p_2}\} \text{ and } \frac{w}{p_1} \neq \frac{w}{p_2}, \\ \frac{w}{p_i} & \text{if } \frac{w}{p_1} = \frac{w}{p_2} \text{ and } i = 1 \\ 0 & \text{otherwise} \end{cases}$$