EconS 526- Homework #3 (Due on November 28th, 2018)

Answer Key

Question #1

A student determined that she has sufficient spare time to attend 24 special events during the school year. Among the events being offered are concerts, hockey games, and theatre productions. She feels the ideal balance would be achieved if she attended twice as many concerts as hockey games and if the number of concerts was equal to the average of the number of hockey games and the number of theatre productions attended. Use Cramer's rule to determine the number of hockey games she should attend to achieve this ideal balance.

Solution

Let us represent the three events as follows:

Concerts: x

Hockey game: y

Theatre production: z

If the student has sufficient spare time to attend 24 special event and among the events are x, y, z, then:

$$x + y + z = 24 \tag{1}$$

If she attended twice as many concerts as hockey games, then:

$$x = 2y \tag{2}$$

If the number of concerts was equal to the average of the number of hockey games and the number of theatre productions attended, then:

$$x = \frac{y+z}{2} \tag{3}$$

Thus, combining the three equation, we have:

$$\begin{aligned} x + y + z &= 24 \\ x - 2y &= 0 \\ 2x - y - z &= 0 \end{aligned} \tag{4}$$

Solving equation (4) using crammer's rule, we rewrite the equation in matrix form:

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -2 & 0 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 24 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

$$AZ = B \tag{6}$$

$$Z_i = \frac{|A_i|}{|A|} \tag{7}$$

$$x=\frac{\begin{vmatrix}24 & 1 & 1\\ 0 & -2 & 0\\ 0 & -1 & -1\end{vmatrix}}{\begin{vmatrix}1 & 1 & 1\\ 1 & -2 & 0\\ 2 & -1 & 1\end{vmatrix}}=\frac{48}{6}=8\ ;\ y=\frac{\begin{vmatrix}1 & 24 & 1\\ 1 & 0 & 0\\ 2 & 0 & -1\end{vmatrix}}{\begin{vmatrix}1 & 1 & 1\\ 1 & -2 & 0\\ 2 & -1 & 1\end{vmatrix}}=\frac{28}{6}=4\ ;\ z=\frac{\begin{vmatrix}1 & 1 & 24\\ 1 & -2 & 0\\ 2 & -1 & 0\end{vmatrix}}{\begin{vmatrix}1 & 1 & 1\\ 1 & -2 & 0\\ 2 & -1 & 1\end{vmatrix}}=\frac{72}{6}=12 \tag{8}$$

Hence, she attends four hockey games to achieve ideal balance.

Question #2

Use Cramer's rule to solve the system of equations

$$\begin{aligned}\frac{4}{7}x-\frac{7}{3}y+\frac{2}{5}z&=\frac{14}{9}\\ -8x+\frac{5}{8}y-6z&=\frac{13}{9}\\ 2x+\frac{3}{5}y+\frac{4}{3}z&=\frac{10}{3}\end{aligned} \tag{9}$$

Round your answer to two decimal places.

Solution

Using basic algebra, we simplify equation (9) as follows:

$$\begin{aligned}180x-725y+126z&=490\\ -576x+45y-432z&=104\\ 90x+27y+60z&=150\end{aligned} \tag{10}$$

Using crammer's rule, we rewrite the equation in matrix form:

$$\begin{pmatrix}180 & -735 & 126\\ -576 & 45 & -432\\ 90 & 27 & 60\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}490\\ 104\\ 150\end{pmatrix} \tag{11}$$

$$AZ=B \tag{12}$$

$$Z_i=\frac{|A_i|}{|A|} \tag{13}$$

$$x=\frac{\begin{vmatrix}490 & -735 & 126\\ 104 & 45 & -432\\ 150 & 27 & 160\end{vmatrix}}{\begin{vmatrix}180 & -735 & 126\\ -576 & 45 & -432\\ 90 & 27 & 160\end{vmatrix}}=\frac{58756068}{3290868}=17.85\ \ y=\frac{\begin{vmatrix}180 & 490 & 126\\ -576 & 104 & -432\\ 90 & 150 & 160\end{vmatrix}}{\begin{vmatrix}180 & -735 & 126\\ -576 & 45 & -432\\ 90 & 27 & 160\end{vmatrix}}=\frac{-1395360}{3290868}=-0.42$$

$$z=\frac{\begin{vmatrix}180 & -735 & 490\\ -576 & 45 & 104\\ 90 & 27 & 150\end{vmatrix}}{\begin{vmatrix}180 & -735 & 126\\ -576 & 45 & -432\\ 90 & 27 & 160\end{vmatrix}}=\frac{-79279020}{3290868}=-24.09 \qquad (14)$$

Question #3 Exercise 10.4 (S&B page 204)

Solution

(a) $\vec{PQ}=(2-0,-1-0)=(2,-1)$

(b) $\vec{PQ}=(1-3,1-2)=(-2,-1)$

(c) $\vec{PQ}=(5-3,3-2)=(2,1)$

(d) $\vec{PQ}=(3-0,1-1)=(3,0)$

(e) $\vec{PQ}=(1-0,2-0,4-0)=(1,2,4)$

(f) $\vec{PQ}=(2-0,-1-1,3-0)=(2,-2,3)$

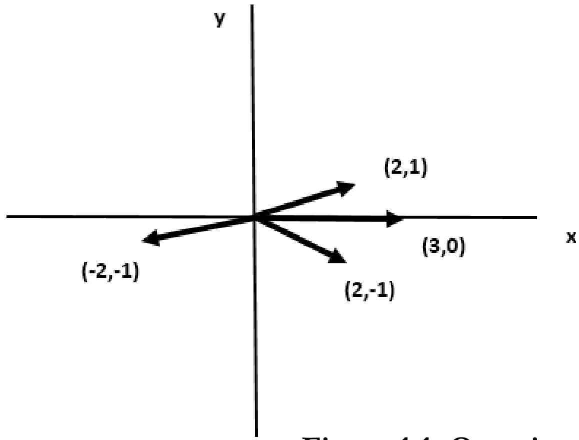

Figure 1.1: Question #3

Question #4 Exercise 10.5 (S&B page 208)

Solution

(a) $u+v=(1+0,2+1)=(1,3)$

(b) $-4w=(-4\times1,-3\times-4)=(-4,12)$

(c) $u+z=(1,2)+(0,1,1)=\text{undefined}$

(d) $3z=(3\times0,3\times1,3\times1)=(0,3,3)$

(e) $2v=(2\times0,2\times1)=(0,2)$

(f) $u+2v=(1+0,2+2)=(1,4)$

(g) $3x+z=(3\times1+0,6+1,0+1)=(3,7,1)$

(h) $2x=(-2\times1,-2\times2,-2\times0)=(-2,-4,0)$

(i) $w+2v=(1,3)+(0,2,2)=\text{undefined}$

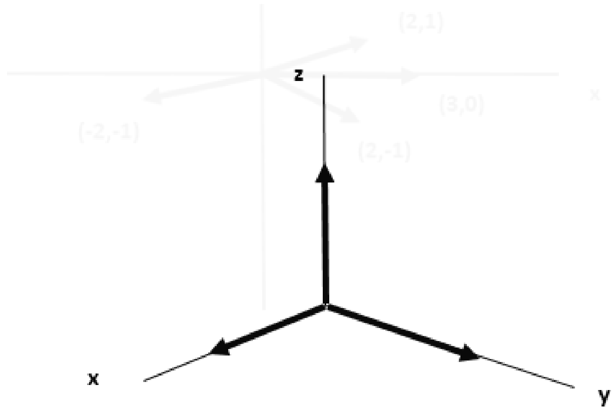

Figure 1.2: Question #3e

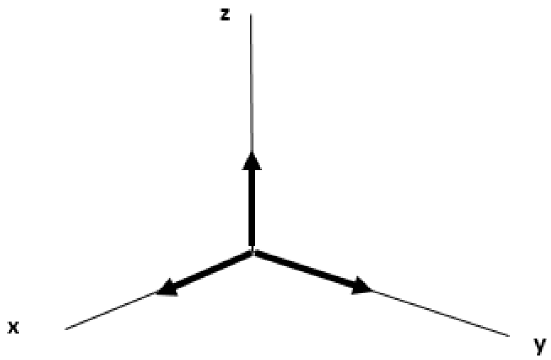

Figure 1.3: Question #3f

Question #5 Exercise 10.11 (S&B page 220)

Solution

(a) $\|PQ\| = \sqrt{(3-0)^2 + (-4-0)^2} = \sqrt{9+16} = \sqrt{25} = 5$

(b) $\|PQ\| = \sqrt{(7-1)^2 + (7+1)^2} = \sqrt{36+64} = \sqrt{100} = 10$

(c) $\|PQ\| = \sqrt{(1-5)^2 + (2-2)^2} = \sqrt{16} = 4$

(d) $\|PQ\| = \sqrt{(2-1)^2 + (-1-1)^2 + (5+1)^2} = \sqrt{1+4+36} = \sqrt{41}$

(e) $\|PQ\| = \sqrt{(1-1)^2 + (0-2)^2 + (-1-3)^2 + (0-4)^2} = \sqrt{0+4+16+16} = \sqrt{36} = 6$

Question #6 Exercise 10.13 (S&B page 220)

(a) Length of the vector x is $\|x\| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$. Therefore,

$$\frac{1}{\|x\|}(3,4) = \frac{1}{5}(3,4) = \left(3/5, 4/5\right)$$

(b) Length of the vector x is $\|x\| = \sqrt{6^2 + 0^2} = \sqrt{36} = 6$. Therefore,

$$\frac{1}{\|x\|}(6,0) = \frac{1}{6}(6,0) = (1,0)$$

(c) Length of the vector x is $\|x\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$. Therefore,

$$\frac{1}{\|x\|}(1,1,1) = \frac{1}{\sqrt{3}}(1,1,1) = \left(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}\right)$$

(d) Length of the vector x is $\|x\| = \sqrt{(-1)^2 + 2^2 + (-3)^2} = \sqrt{1+4+9} = \sqrt{14}$. Therefore,

$$\frac{1}{\|x\|}(-1,2,-3) = \frac{1}{\sqrt{14}}(-1,2,-3) = \left(-1/\sqrt{14}, 2/\sqrt{14}, -3/\sqrt{14}\right)$$