Final Exam - EconS 526

December 13th, 2018

NAME: ____________________

Instructions. *Show all your calculations (step by step).*

Question #1 (20 Points). Use the inner product to compute the angle θ between the diagonal d of the cube and a (see Figure 1).

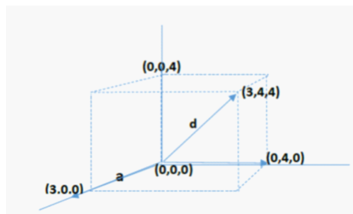

Figure 1

Solution

The diagonal d is the vector $(3,4,4)$. The angle θ between a and d satisfies

$$\begin{aligned}\cos\theta &= \frac{\mathbf{a}\cdot\mathbf{d}}{\|\mathbf{a}\|\,\|\mathbf{d}\|}\\ &= \frac{(3,0,0)(3,4,4)}{3\sqrt{9+16+16}} = \frac{9}{3\sqrt{41}} = \frac{3}{\sqrt{41}}\end{aligned}$$

Hence, $\theta \simeq 62^0$

Question #2 (20 Points). (a) Prove that for any two vectors $\mathbf{z}$ and $\mathbf{w}$ in $\mathbb{R}^n$,

$$\|\mathbf{z}+\mathbf{w}\| \leq \|\mathbf{z}\| + \|\mathbf{w}\|.$$

(b) Consider a line that goes through $(4,3)$ and moves to the north east in direction $(1,2)$. Described the parameterization $(x_1(t), x_2(t))$ of the line and show that your result coincides with the equation of the line when you know two points: $(2,-1)$ and $(0,-5)$.

Solution

(a) Recall that $\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \cos\theta \leq 1$ then

$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &\leq \|\mathbf{u}\|\,\|\mathbf{v}\|\\ \|\mathbf{u}\|^2 + 2(\mathbf{u}\cdot\mathbf{v}) + \|\mathbf{v}\|^2 &\leq \|\mathbf{u}\|^2 + 2\|\mathbf{u}\|\,\|\mathbf{v}\| + \|\mathbf{v}\|^2\\ \mathbf{u}\cdot\mathbf{u} + \mathbf{u}\cdot\mathbf{v} + \mathbf{u}\cdot\mathbf{v} + \mathbf{v}\cdot\mathbf{v} &\leq (\|\mathbf{u}\| + \|\mathbf{v}\|)^2\\ (\mathbf{u}+\mathbf{v})(\mathbf{u}+\mathbf{v}) &\leq (\|\mathbf{u}\| + \|\mathbf{v}\|)^2\\ \|\mathbf{u}+\mathbf{v}\|^2 &\leq (\|\mathbf{u}\| + \|\mathbf{v}\|)^2\\ \|\mathbf{u}+\mathbf{v}\| &\leq \|\mathbf{u}\| + \|\mathbf{v}\|\end{aligned}$$

(b)

$$\begin{aligned}x(t) &= (4,3) + t(1,2)\\ &= (4+t, 3+2t)\\ x_1 &= 4+t\\ x_2 &= 3+2t\end{aligned}$$

hence, the equation of the line is

$$\begin{aligned}\frac{x_2-3}{2} &= x_1 - 4\\ x_2 &= 2x_1 - 5\end{aligned}$$

If we consider the equation of the line when we know points $(2, -1)$ and $(0, -5)$

$$\begin{aligned} x_2 - b &= \frac{d-b}{c-a}(x_1 - a) \\ x_2 + 1 &= \frac{-5+1}{0-2}(x_1 - 2) \\ x_2 &= 2x_1 - 5 \end{aligned}$$

Question #3 (20 Points). Let us consider the IS/LM model. That model, represented in matrix format and in changes rather than levels, is

$$H \cdot \Delta x = \Delta y,$$

or more explicitly,

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ -0.8 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2000 \\ 0.1 & 0 & 0 & -1000 \end{bmatrix} \begin{bmatrix} \Delta Y \\ \Delta C \\ \Delta I \\ \Delta R \end{bmatrix} = \begin{bmatrix} \Delta G \\ 0 \\ 0 \\ \Delta \frac{M}{P} \end{bmatrix}$$

where the changes in consumption (ΔC), investment (ΔI), income (ΔY), and interest rate(ΔR) are endogeneous, while the changes in government spending (ΔG) and the real money supply ($\Delta \frac{M}{P}$) are exogenous. The first row H represents the national income accounting identity, the second row represents the consumption equation, the third row represents the investment demand equation, and the fourth row represents the money market equilibrium equation.

(a) Using Cramer's rule determine the effect of an expansion of government spending by \$100 on income, *ceteris paribus [Hint: consider that* $\Delta \frac{M}{P} = 0$*]*. Discuss your result.

Solution

(a)Replacing the first column of H as appropriate for this question, we have

$$H_1 = \begin{bmatrix} 100 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2000 \\ 0 & 0 & 0 & -1000 \end{bmatrix}$$

and expanding along the first column, we find

$$\begin{aligned} |H_1| &= 100(-1)2 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 2000 \\ 0 & 0 & -1000 \end{vmatrix} \\ &= 100(-1000 \times 1 - 2000 \times 0) = -100,000 \end{aligned}$$

In addition, $|H| = -400$, and hence

$$\Delta Y = \frac{|H_1|}{|H|} = \frac{-100,000}{-400} = 250$$

This result shows that an increase in government spending by \$100 results in an overall increase in income by \$250.