Recitation 12 - Friday November 30th

1. Assume that Alaska Airlines is a monopolist in the route Pullman-Seattle. The Washington State Legislature would like to design a policy that induces Alaska Airlines to voluntarily produce an efficient output level. If the firm faces an inverse demand function $p(q)$ with $p'(q) < 0$ and marginal costs $c'(q) > 0$ for all q, show that such a policy must be a subsidy, and determine the exact amount of the subsidy.

   - Let t be the tax/subsidy per unit of output. Then the monopolist maximizes

$$\max_{q \geq 0} p(q)q - c(q) - tq$$

where, at this point, for generality we allow t to be positive (for taxes) or negative (for subsidies). Taking first order condition with respect to q yields

$$p'(q)q + p(q) - c'(q) - t \leq 0, \text{ with equality if } q > 0$$

Solving for t (and assuming an interior solution) we obtain

$$t = p'(q)q + p(q) - c'(q) \quad (1)$$

Since we seek to induce an efficient outcome, the monopolist must choose a level of output q that satisfies $p(q) = c'(q)$. This implies that the tax must be $t = p'(q)q$, so that the monopolist's profit-maximizing condition (1) becomes

$$\underbrace{p'(q)q}_{t} = p'(q)q + p(q) - c'(q)$$

which simplifies into the efficient outcome condition

$$p(q) = c'(q)$$

However, we know that $t = p'(q)q < 0$ since $p'(q) < 0$. Therefore, the tax that the government imposes on the monopoly is actually a "negative tax," i.e., a subsidy. Intuitively, the government must subsidize the monopolist in order to induce a larger level of production.

1. - *Parametric example.* Consider, for instance, that the inverse demand the firm faces is $p(q) = a - bq$ and that the cost function is $c(p) = F + cq$, where F denotes a fixed cost, where $a > c > 0$. The optimal output level q^* in this context satisfies

$p(q) = c'(q)$, or $a - bq^* = c$, which yields an output level of $q^* = \frac{a-c}{b}$. In contrast, the monopolist produces $q^m = \frac{a-c}{2b}$. Therefore, the subsidy would need to be

$$t = p'(q^*)q^* = -b\frac{a-c}{b} = c - a\,,$$

which is a negative tax since $a > c > 0$ holds by definition. Alternatively, a subsidy of $a-c$ provides the monopolist with the incentives to produce the socially optimal output $q^* = \frac{a-c}{b}$. Indeed, if we add this subsidy into the monopolist's profit-maximization problem, we obtain

$$\max_{q \geq 0} p(q)q - c(q) + (a-c)q$$

Taking first-order conditions with respect to q, yields

$$a - 2bq^m - c + (a-c) = 0$$

which further simplifies to $a - c = bq^m$. Solving for q, we obtain a monopoly output of $q^m = \frac{a-c}{b}$, which exactly coincides with the socially optimal output $q^* = \frac{a-c}{b}$ the regulator sought to induce.

2. Consider a monopolist selling two different goods, q_1 and q_2, whose demands are interdependent and given by

$$q_i = a - bp_i + gp_j \quad \text{for } i = \{1, 2\} \text{ and } j \neq i$$

where $b > 0$ (this guarantees that the demand for a particular good decreases in its own price). In addition, note that if $g > 0$, goods are substitutes, while if $g < 0$, they are complements (and if $g = 0$ the products are independent of each other). Assume also that $|g| < b$, which guarantees that the own-price effect, represented in parameter b, dominates the cross-price effect, embodied in parameter g. Intuitively, the demand for good i is more sensitive to a given increase in its own price than to the same increase in the price of a different good. Finally, consider that $a > c(b-g)$ which guarantees that output is strictly positive in equilibrium. In order to focus on the interdependence between both products' demands, let us assume that the marginal costs of production coincide across both products. That is, total costs are $TC(q_1, q_2) = cq_1 + cq_2$.

   (a) Find the profit-maximizing price for good 1, p_1, and for good 2, p_2, for this monopolist. [*Hint:* Find the profit-maximizing prices rather than output levels.]

      - The monopolist profit maximization problem is to choose prices p_1 and p_2

that solve

$$\max_{p_1, p_2} \ \pi = (a - bp_1 + gp_2)(p_1 - c) + (a - bp_2 + gp_1)(p_2 - c).$$

Taking first order condition respect to every price p_i, yields

$$\frac{\partial \pi}{\partial p_i} = a - 2bp_i + 2gp_j + c(b - g) = 0,$$

for every product $i, j = \{1, 2\}$ and $i \neq j$. At the symmetric solution $p_1 = p_2 = p_m$, which implies

$$a - 2bp_m + 2gp_m + c(b - g) = 0,$$

and solving for p_m, we obtain

$$p_m = \frac{a + c(b - g)}{2(b - g)},$$

where $p_m > c$. In particular, recall that $a > c(b - g)$ holds by assumption, which implies that, if we add $c(b - g)$ on both sides of this inequality, we obtain

$$a + c(b - g) > 2c(b - g),$$

which must also hold. The last condition can alternatively be expressed as

$$\frac{a + c(b - g)}{2(b - g)} > c$$

thus entailing that $p_m > c$.

(b) Do prices increase or decrease in the parameter that reflects the cross-price effects, g?

- Taking the first order derivative of p_m with respect to g, yields,

$$\frac{\partial p_m}{\partial g} = \frac{a}{2(b - g)^2} > 0$$

implying that as g increases in the interval $(-b, b)$, the price charged by the monopolist on both products also increases. [It is easy to check that p_m is concave in g, with its lowest possible value being $\frac{a+2bc}{4b}$, which occurs when $g \to -b$, and an asymptote as $g \to b$.]

- For illustrative purposes, figure 1 assumes parameters $a = 1$, $b = 1$ and $c = 0$, implying a price of $p_m = \frac{1}{2(1-g)}$, and allows for g to take values in the interval $(-b, b)$, i.e., $(-1, 1)$.

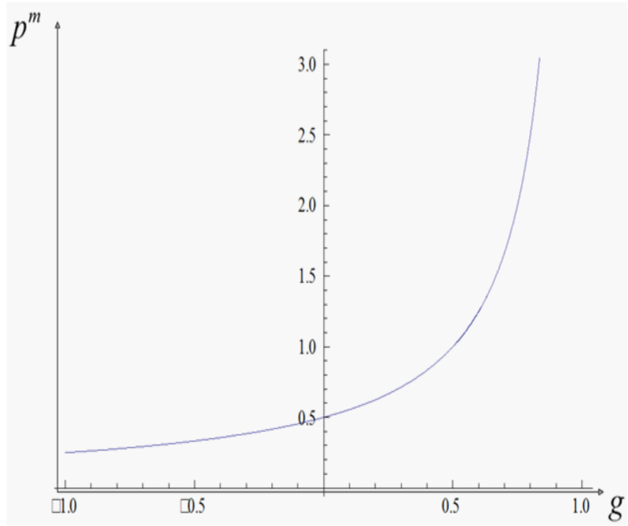

Figure 1. Monopoly price p_m increases in g.

(c) Compare prices if $g > 0$ and if $g < 0$ with those where $g = 0$. Explain

- *Complements.* Relative to the benchmark case where the two products are independent ($g = 0$), the monopolist reduces the price of its product when they are complements ($g < 0$), and it increases it when they are substitutes ($g > 0$). The intuition for this result is straightforward. When products are complements, they exercise a positive effect on each other and the monopolist internalizes such effect by decreasing its prices (i.e., a lower price of good 1 stimulates sales of good 2, and vice versa). In other words, if products 1 and 2 were sold by two distinct monopolists, consumers would pay more for them than when they are sold by the same firm. [This is a standard result in the literature on vertically integrated firms studied in industrial organization].
- *Substitutes.* When, instead, products are substitutes ($g > 0$), the external effect they exercise on each others' demands is negative, and the monopolist controls it by raising prices (a lower price of good 1 crowds out sales of good 2 and vice versa). Hence, if products 1 and 2 were sold by two distinct firms, consumers would pay less than when they are sold by the same firm.
- *Remark:* note that these insights extend to the case where the monopolist sells the same product in sequential markets, i.e., to a group of customers in the first period and to a different set of customers in the second period. In particular, we can re-interpret the demand relationship as one of intertem-

poral substitutability and complementarity, i.e., demand in the first period stimulates (reduces) demand in the second period when $g > 0$ ($g < 0$, respectively).