

Quiz #3 - EconS 527
November 14th, 2018

Question #1 (50 Points)

Find a nonzero 2×2 matrix A such that $A^2 = 0$

Solution

For instance matrix

$$\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \times \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

More generally, any matrix where $a \in \mathbb{R}$

$$\begin{pmatrix} a & -a \\ a & -a \end{pmatrix} \times \begin{pmatrix} a & -a \\ a & -a \end{pmatrix} = \begin{pmatrix} a^2 - a^2 & -a^2 + a^2 \\ a^2 - a^2 & -a^2 + a^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Question #2 (50 Points)

Inver the matrices

$$A = \begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 1 & 1 \\ 3 & 1 & 2 \end{pmatrix}$$

Solution

$$A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 1 & 1 \\ -2 & 3 \end{pmatrix}$$

$$B = \left(\begin{array}{ccc|ccc} 2 & 1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 1 & 0 \\ 3 & 1 & 2 & 0 & 0 & 1 \end{array} \right)$$

Starting with the augmented matrix, we subtract $3/2$ times the first row from each of the other rows to get

$$\left(\begin{array}{ccc|ccc} 2 & 1 & 2 & 1 & 0 & 0 \\ 0 & -\frac{1}{2} & -2 & -\frac{3}{2} & 1 & 0 \\ 0 & -\frac{1}{2} & -1 & -\frac{3}{2} & 0 & 1 \end{array} \right)$$

Next add twice the second row to the first row, and subtract the second row from the third row. We obtain

$$\left(\begin{array}{ccc|ccc} 2 & 0 & -2 & -2 & 2 & 0 \\ 0 & -\frac{1}{2} & -2 & -\frac{3}{2} & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array} \right)$$

one more elimination step gives

$$\left(\begin{array}{ccc|ccc} 2 & 0 & 0 & -2 & 0 & 2 \\ 0 & -\frac{1}{2} & 0 & -\frac{3}{2} & -1 & 2 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array} \right)$$

To obtain B^{-1} we divide the first row by 2 and the second row by $-1/2$

$$B^{-1} = \begin{pmatrix} -1 & 0 & 1 \\ 3 & 2 & -4 \\ 0 & -1 & 1 \end{pmatrix}$$