Midterm #2 - EconS 526

November 26th, 2018

NAME: ____________________

Instructions. *Show all your calculations (step by step).*

Question #1 (20 Points)

Show that Cramer's rule can be used to solve the following system, and then use it to find the value of x that satisfies the system:

$$\begin{cases} x+3y+2w+1=z \\ 2w+z=2x+4y+1 \\ x+2y+3z+3w-3=0 \\ 2x+7y+6z+2w=6 \end{cases}$$

Solution

In order to use cramer's rule we need to identify the coefficient matrix A, such that $Ax=b$. In this case

$$A=\begin{pmatrix} 1 & 3 & 2 & -1 \\ -2 & -4 & 2 & 1 \\ 1 & 2 & 3 & 3 \\ 2 & 7 & 2 & 6 \end{pmatrix}, b=\begin{pmatrix} -1 \\ 1 \\ 3 \\ 6 \end{pmatrix} \text{ and } x=\begin{pmatrix} x \\ y \\ w \\ z \end{pmatrix}$$

Hence, using Cramer's rule we have that in order to obtain x then we use $x=\frac{\det B_x}{\det A}$. The $\det A$ is 103 and $\det B_x$ is 0, hence, $x=\frac{0}{103}=0$ (note that in order to calculate the determinant of a $4x4$ matrix you will need to use the cofactor, $\det A = a_{11}C_{11}+a_{12}C_{12}+a_{13}C_{13}+a_{14}C_{14}$)

Question #2 (20 Points)

Solve the given system by using the inverse of the coefficient matrix

$$\begin{cases} 3x+y+4z=1 \\ x+z=0 \\ 2y+z=2 \end{cases}$$

Solution

The coefficient matrix is

$$A=\begin{pmatrix} 3 & 1 & 4 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

Hence, $A^{-1}=\frac{1}{\det A} Adj A$. The $\det A$ is 1, and the $Adj A$ is

$$Adj A=\begin{pmatrix} -2 & 7 & 1 \\ -1 & 3 & 1 \\ 2 & -6 & -1 \end{pmatrix}$$

As a consequence,

$$A^{-1}=\begin{pmatrix} -2 & 7 & 1 \\ -1 & 3 & 1 \\ 2 & -6 & -1 \end{pmatrix}$$

and $Ax=b$, implies $x=A^{-1}b$

$$x=\begin{pmatrix} -2 & 7 & 1 \\ -1 & 3 & 1 \\ 2 & -6 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}=\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Question #3 (20 Points)

Determine whether the following matrices are singular or nonsingular:

$$\begin{pmatrix} 1 & 0 & -2 \\ -1 & 2 & 3 \\ 0 & 2 & 1 \end{pmatrix} \text{ and } \begin{pmatrix} 2 & -1 & 1 \\ 1 & 0 & 2 \\ -1 & 1 & 3 \end{pmatrix}$$

Solution

Let us calculate the rank. In the first matrix the rank is 2 and hence rank$\neq$number of rows, therefore, it is Singular. In the second matrix the rank is 3 and it coincides with the number of rows, hence, it is nonsingular.

Question #4 (20 Points)

Consider the following system and, *without solving it*, discuss whether or not it has a unique solution.

$$\begin{aligned} x + 4y + 7z &= 2 \\ 2x + 5y + 8z &= 1 \\ 3x + 6y + 9z &= -1 \end{aligned}$$

Solution

It does not have a unique solution since the coefficient matrix has a rank equal to 2.

$$\begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

Question #5 (10 Points)

Let us consider the following matrix

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

Please write each expression: (1) the minor of a_{32} and (2) the cofactor of a_{13}.

Solution

$$\begin{pmatrix} a_{11} & a_{13} & a_{14} \\ a_{21} & a_{23} & a_{24} \\ a_{41} & a_{43} & a_{44} \end{pmatrix}$$

$$\begin{pmatrix} a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \\ a_{41} & a_{42} & a_{44} \end{pmatrix}$$

Question #6 (10 Points)

Compute the required matrix if

$$A = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

when:

(i) $(2A)^T - 3I^2$

(ii) $B^4 + I^4$

Solution

(i) $\begin{pmatrix} -1 & -2 \\ 2 & 1 \end{pmatrix}$

(ii) $\begin{pmatrix} 2 & 0 \\ 0 & 17 \end{pmatrix}$