Homework # 8 - [Due on November 16th, 2018]

1. Consider an industry with two polluting firms, but where one of them produces more environmental damage per unit of output (labelled as the brown firm) than its rival (green firm). Firm i's marginal production costs are given by c_i where $i = \{B, G\}$, where c_B is allowed to be higher or lower than c_G. In addition, the social welfare function that the regulator uses to set emission fees on these firms is

$$SW = CS + PS + T - Env$$

where CS is the consumer surplus, PS is the producer surplus, $T = t_B q_B + t_G q_G$ is the tax revenue from emission fees on both firms, and $Env = d_B(q_B)^2 + d_G(q_G)^2$ is the environmental damage from the production of the brown and green firms, where $d_B \geq d_G > \frac{1}{2}$. Finally, assume that firms sell differentiated products. In particular, the inverse demand function of firm $i = \{B, G\}$ is

$$p_i(q_i, q_j) = 1 - q_i - gq_j \text{ where } j = \{B, G\} \text{ and } j \neq i.$$

Hence, parameter $g \in [0, 1]$ represents the degree of product differentiation, where if $g = 0$ firms sell completely differentiated products (and, hence, their demands are independent of each other), i.e., $p_i = 1 - q_i$ for every firm i; while if $g = 1$, firms sell homogeneous products, yielding an inverse demand function $p_i = 1 - q_i - q_j = 1 - Q$, where $Q = q_i + q_j$.

   (a) Show that consumer surplus in this market is given by $CS = \frac{1}{2}(q_B^2 + 2gq_Bq_G + q_G^2)$. Interpret.

   (b) *No regulation.* Find equilibrium output levels when firms do not face emission fees. Evaluate these output levels in the extreme cases of $g = 0$ and $g = 1$. Interpret.

   (c) *Regulation.* Find equilibrium output levels when firms face any emission fee t. Evaluate these output levels in the extreme cases of $g = 0$ and $g = 1$. Interpret.

   (d) Identify the socially optimal output level for the brown firm, q_B^{SO}, and for the green firm, q_G^{SO}.

   (e) Find the socially optimal fees (t_B, t_G) that induce firms to produce the socially optimal output levels found in part (d).

2. Consider two consumers with utility functions over two goods, x_1 and x_2, given by

$$\begin{aligned} u_A &= \log(x_1^A) + x_2^A - \frac{1}{2}\log(x_1^B) \quad \text{for consumer } A\text{, and} \\ u_B &= \log(x_1^B) + x_2^B - \frac{1}{2}\log(x_1^A) \quad \text{for consumer } B. \end{aligned}$$

where the consumption of good 1 by individual $i = \{A, B\}$ creates a negative externality on individual $j \neq i$ (see the third term, which enters negatively on each individual's utility function). For simplicity, consider that both individuals have the same wealth, m, and that the price for both goods is 1.

(a) *Unregulated equilibrium.* Set up consumer A's utility maximization problem, and determine his demand for goods 1 and 2, as x_1^A and x_2^A. Then operate similarly to find consumer B's demand for good 1 and 2, as x_1^B and x_2^B.

(b) *Social optimum.* Calculate the socially optimal amounts of x_1^A, x_2^A, x_1^B and x_2^B, considering that the social planner maximizes a utilitarian social welfare function, namely, $W = U_A + U_B$.

(c) *Restoring efficiency.* Show that the social optimum you found in part (b) can be induced by a tax on good 1 (so the after-tax price becomes $1+t$) with the revenue returned equally to both consumers in a lump-sum transfer.