Homework # 7 - [Due on November 2nd, 2018]

1. Consider a perfectly competitive industry with N symmetric firms, each with cost function $c(q) = F + cq$, where $F, c > 0$. Assume that the inverse demand is given by $p(Q) = a - bQ$, where $a > c$, $b > 0$, and where Q denotes aggregate output.

   (a) *Short-run equilibrium.* If exit and entry is not possible in the industry (assuming N firms remain active), find the individual production level of each firm.

   - Each invidual firm i solves the PMP

   $$\max_{q_i \geq 0} (a - bQ)q_i - (F + cq_i) = \left(a - bq_i - b\sum_{j \neq i} q_j\right) q_i - (F + cq_i)$$

     Taking first-order conditions with respect to q_i yields

   $$a - 2bq_i - b\sum_{j \neq i} q_j - c = 0$$

     and applying symmetry in equilibrium outputs, i.e., $q_1 = q_2 = ... = q_N$, we obtain an individual equilibrium output

   $$q_i = \frac{a - c}{b(N+1)}$$

     for every firm $i \in N$. Note that this result is a function of the number of active firms in the industry, N.

   - In this setting, the equilibrium market price is

   $$p^* = a - b\underbrace{\left(N \cdot \frac{a - c}{b(N+1)}\right)}_{Q = N \cdot q_i} = \frac{a + Nc}{N + 1}$$

   (b) *Long-run equilibrium.* Consider now that firms have enough time to enter the industry (if economic profits can be made) or to exit (if they make losses by staying in the industry). Find the long-run equilibrium number of firms in this perfectly competitive market.

   - In a long-run equilibrium of a perfectly competitive market, we know that firms must be making no economic profits, $\pi = 0$, as otherwise firms would still have incentives to enter or exit the industry. Hence, we first need to find the equilibrium profits that every individual firm i earns by producing the equilibrium output q_i found in part (a). In particular, these profits are

$$\pi_i = (a - b\underbrace{Nq_i}_{Q})q_i - (F + cq_i) = \frac{(a-c)^2}{b(N+1)^2} - F$$

setting them equal to zero and solving for N yields the long-run equilibrium number of firms, $\lfloor N^* \rfloor = \frac{a-c}{\sqrt{bF}} - 1$, where $\lfloor N \rfloor$ indicates the highest integer smaller or equal to N. For instance, if $a = b = 1$, $c = \frac{1}{4}$, and $F = \frac{1}{16}$ N^* becomes $N^* = 2$.

- More generally, note that the expression we found for equilibrium profits, π_i, is monotonically decreasing in the number of firms, N, for all parameter values, that is

$$\frac{\partial \pi_i}{\partial N} = -2b(a-c)^2(N-1) < 0$$

thus implying that equilibrium profits becomes zero for a sufficiently large number of firms.

2. A tax is to be levied on a commodity bought and sold in a competitive market. Two possible forms of tax may be used: In one case, a *per unit* tax is levied, where an amount t is paid per unit bought or sold. In the other case, an *ad valorem* tax is levied, where the government collects a tax equal to τ times the amount the seller receives from the buyer. Assume that a partial equilibrium approach is valid.

   (a) Show that, with a per unit tax, the ultimate cost of the good to consumers and the amounts purchased are independent of whether the consumers or the producers pay the tax. As a guidance, let us use the following steps:

   1. *Consumers:* Let p^c be the competitive equilibrium price when the *consumer* pays the tax. Note that when the consumer pays the tax, he pays $p^c + t$ whereas the producer receives p^c. State the equality of the (generic) demand and supply functions in the equilibrium of this competitive market when the consumer pays the tax.

      - If the per unit tax t is levied on the consumer, then he pays $p+t$ for every unit of the good, and the demand at market price p becomes $x(p+t)$. The equilibrium market price p^c is determinded from equalizing demand and supply:

      $$x(p^c + t) = q(p^c).$$

   2. *Producers:* Let p^p be the competitive equilibrium price when the *producer* pays the tax. Note that when the producer pays the tax, he receives $p^p - t$

whereas the consumer pays p^p. State the equality of the (generic) demand and supply functions in the equilibrium of this competitive market when the producer pays the tax.

- On the other hand, if the per unit tax t is levied on the producer, then he collects $p - t$ from every unit of the good sold, and the supply at market price p becomes $q(p - t)$. The equilibrium market price p^p is determined from equalizing demand and supply:

$$x(p^p) = q(p^p - t).$$

(b) Show that if an equilibrium price p solves your equality in part (a), then $p + t$ solves the equality in (b). Show that, as a consequence, equilibrium amounts are independent of whether consumers or producers pay the tax.

- It is easy to see that p solves the first equation if and only if $p + t$ solves the second one. Therefore, $p^p = p^c + t$, which is the ultimate cost of the good to consumers in both cases. The amount purchased in both cases is

$$x(p^p) = x(p^c + t).$$

(c) Show that the result in part (b) is not generally true with an ad valorem tax. In this case, which collection method leads to a higher cost to consumers? [*Hint:* Use the same steps as above, first for the consumer and then for the producer, but taking into account that now the tax increases the price to $(1+\tau)p$. Then, construct the excess demand function for the case of the consumer and the producer.]

- If the ad valorem tax τ is levied on the consumer, then he pays $(1+\tau)p$ for every unit of the good, and the demand at market price p becomes $x((1+\tau)p)$. The equilibrium market price p^c is determined from equalizing demand and supply:

$$x((1+\tau)p^c) = q(p^c).$$

On the other hand, if the ad valorem tax τ is levied on the producer, he receives $(1+\tau)p$ for every unit of the good sold, and the supply at market price p becomes $q((1-\tau)p)$. The equilibrium market price p^p is determined from equalizing demand and supply:

$$x(p^p) = q((1-\tau)p^p).$$

Consider the excess demand function for this case:

$$z(p) = x(p) - q((1-\tau)p) \tag{1}$$

Since the demand curve $x(\cdot)$ is non-increasing and the supply curve $q(\cdot)$ is non-decreasing, $z(p)$ must be non-increasing. From (1) we have

$$\begin{aligned} z((1+\tau)p^c) &= x((1+\tau)p^c) - q((1-\tau)[(1+\tau)p^c]) = \\ &= x((1+\tau)p^c) - q((1-\tau^2)p^c) \geq \\ &\geq x((1+\tau)p^c) - q(p^c) = 0, \end{aligned}$$

where the inequality takes into account that $q(\cdot)$ is non-decreasing.

- Therefore, $z((1+\tau)p^c) \geq 0$ and $z(p^p) = 0$. Since $z(\cdot)$ is non-increasing, this implies that $(1+\tau)p^c \leq p^p$. In words, levying the ad valorem tax on consumers leads to a lower cost on consumers than levying the same tax on producers. (In the same way, it can be shown that levying the ad valorem tax on consumers leads to a higher price for producers than levying the same tax on producers).

(d) Are there any special cases in which the collection method is irrelevant with an ad valorem tax? [*Hint:* Think about cases in which the tax introduces the same wedge on consumers and producers (inelasticity). Then prove your statement by using the above argument on excess demand functions.]

- If the supply function $q(\cdot)$ is strictly increasing, the argument can be strengthened to obtain the strict inequality: $(1+\tau)p^c < p^p$. On the other hand, when the supply is perfectly inelastic, i.e., $q(p) = \overline{q}$ =constant, then yield

$$x((1+\tau)p^c) = \overline{q} = x(p^p),$$

and therefore $p^p = (1+\tau)p^c$. Here both taxes result in the same cost to consumers. However, producers still bear a higher burden when the tax is levied directly on them:

$$(1-\tau)p^p = (1-\tau)(1+\tau)p^c < p^c.$$

these prices are depicted in the next figure, where $x(p)$ reflects the demand function with no taxes and $x((1-\tau)p)$ represents the demand function with the ad valorem tax. While the inelastic supply curve guarantees that sales are unaffected by the tax (remaining at $\overline{q}$ units), the price that the producer receives drops from p^p to $(1-\tau)p^p$. Therefore, the two taxes are still not fully equivalent.

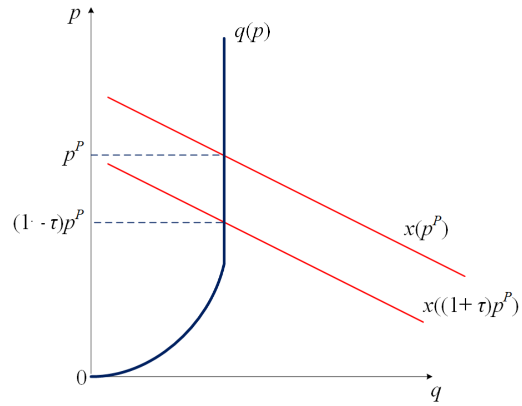

Figure 1. Introducing a tax.

- The intuition behind these results is simple: with a tax, there is always a wedge between the "consumer price"and the "producer price." Levying an ad valorem tax on the producer price, therefore, results in a higher tax burden (and a higher tax revenue) than levying the same percentage tax on consumers.

3. Consider a national government seeking to regulate a polluting industry with firms operating in N regions (i.e., states or provinces). The cost of emission reduction in county i is $C_i(e_i)$, which is decreasing and convex in emissions, e_i. The environmental damage from emissions in region i is denoted by $D_i(e_i, E)$, which is increasing and convex in this region's emissions, e_i, and in aggregate emissions, $E = e_1 + e_2 + ... + e_N$, thus reflecting the presence of negative externalities of emissions across jurisdictions.

   (a) *First-best policy (regionally differentiated fees).* Assume that the national government seeks to minimize the sum of aggregate costs of emission reduction along with aggregate environmental damages. (This is analogous to maximizing social welfare when output and emissions are unrelated, since both $C_i(\cdot)$ and $D_i(\cdot)$ enter negatively in the welfare function.) Find the emission fee that leads to the first-best level of emissions. Interpret.

   - The government chooses emissions at region i, e_i, to solve

$$\min_{e_i} \sum_{i=1}^{N} C_i(e_i) + D_i(e_i, E)$$

taking first-order conditions with respect to e_i yields

$$\underbrace{-\frac{\partial C_i(e_i)}{\partial e_i}}_{\text{MC of emission reduction}} = \underbrace{\frac{\partial D_i(e_i, E)}{\partial e_i}}_{\text{Mg Damage at } i} + \underbrace{\sum_j \frac{\partial D_j(e_j, E)}{\partial E}}_{\text{Mg Damage at } j} \tag{1}$$

since $E = e_1 + e_2 + ... + e_N$. In words, optimal emissions in each region i entail that the marginal cost of emissions reductions in that region (represented in the left-hand side of the above expression) coincides with the marginal damage from pollution emissions in region i plus the transboundary effects of its emissions on all other regions (as captured in the right-hand side of expression 1). The first best regulation can be implemented through differentiated emissions quotas per region set at the level defined by equation (1).

- Alternatively, the first-best outcome can be achieved by differentiated emission fees t_i equal to the marginal damages identified in the right-hand side of expression (1):

$$t_i = \frac{\partial D_i(e_i, E)}{\partial e_i} + \sum_j \frac{\partial D_j(e_j, E)}{\partial E} \tag{2}$$

Each region i pays a fee equal to the sum of the marginal damage from pollution emissions in region i and the transboundary effects of its emissions on all regions. Local regulations will be more stringent in those regions where the local damages of pollution are larger and for those regions causing larger transboundary effects on other regions. However, differentiated first-best policies (such as differented emission fees) might not always be politically feasible since they could be challenged in court as discriminatory.

(b) *Second-best policy (uniform fees).* Most emission fees in real life are not spatially differentiated. Assume that the national government imposes a uniform emission fee T on all regions. Find this emission fee and show that it can be expressed as the average environmental damage in all regions.

- The national regulator solves the following problem:

$$\min_{e_i} \sum_{i=1}^{N} C_i(e_i(T)) + D_i(e_i(T), E(T))$$

subject to firms' reaction function:

$$-\frac{\partial C_i(e_i)}{\partial e_i} = T \quad \text{for every region } i. \tag{3}$$

In words, this problem says that the regulator sets a national policy anticipating how firms will respond to its regulation (i.e., reducing emission until the point where the additional cost of emission reduction coincides with the fee the firm would have to pay for that unit of emission).

- Differentiating the above objective function with respect to e_i, we find

$$\sum_{i=1}^{N}\left(-\frac{\partial C_i(e_i)}{\partial e_i}\right)=\sum_{i=1}^{N}\left(\frac{\partial D_i(e_i,E)}{\partial e_i}+\sum_j\frac{\partial D_j(e_j,E)}{\partial E}\right)$$

which can be rearranged as

$$\sum_{i=1}^{N}\left(-\frac{\partial C_i(e_i)}{\partial e_i}\right)=\sum_{i=1}^{N}\left(\frac{\partial D_i(e_i,E)}{\partial e_i}\right)+N\sum_j\frac{\partial D_j(e_j,E)}{\partial E} \tag{4}$$

Recall the firms' response function in equation (3), $-\frac{\partial C_i(e_i)}{\partial e_i}=T$, which aggregating over all N firms yields

$$\sum_{i=1}^{N}\left(-\frac{\partial C_i(e_i)}{\partial e_i}\right)=NT.$$

Since this coincides with the left-hand side of expression (4), we can rewrite (4) as follows

$$NT=\sum_{i=1}^{N}\left(\frac{\partial D_i(e_i,E)}{\partial e_i}\right)+N\sum_j\frac{\partial D_j(e_j,E)}{\partial E}$$

or, dividing both sides by N, as follows

$$T=\frac{1}{N}\sum_{i=1}^{N}\frac{\partial D_i(e_i,E)}{\partial e_i}+\sum_j\frac{\partial D_j(e_j,E)}{\partial E} \tag{5}$$

Intuitively, the uniform emission fee T coincides with the average marginal damage across regions.

(c) *Comparison.* Under which case the uniform emission fee you found in part (b) coincides with that found in part (a), thus becoming socially optimal?

- Comparing equations (2) and (5), we see that the uniform emission fee is only optimal when the marginal damage from pollution is the same for all regions (e.g., states or provinces). Otherwise, the uniform emission fee will not yield the first-best solution in part (a), namely, the uniform emission fee would be too low for those regions where local damages are high and too stringent for those regions where local damages are low.