Solutions for EconS 527 – Homework #1

1. Exercise 6.3 and 6.6 from Chapter 6 in S&B (page 121)

Exercise 6.3

$x_1 = 0.5x_1 + 0.5x_2 + 1$, $x_2 = 0x_1 + 0.25x_2 + 3$, the solution is $x_1 = 6, x_2 = 4$

Exercise 6.6

For black females, $\begin{cases} x_{t+1} = 0.993x_t + 0.106y_t \\ y_{t+1} = 0.007x_t + 0.894y_t \end{cases}$

To find the stationary distribution, set $x_{t+1} = x_t = x$ and $y_{t+1=}y_t = y$: $x = 0.9381$ and $y = 0.0619$

For white females, $\begin{cases} x_{t+1} = 0.997x_t + 0.151y_t \\ y_{t+1} = 0.003x_t + 0.849y_t \end{cases}$

Stationary solution: $x = 0.9805$ and $y = 0.0195$

2. Exercise 7.16 and 7.18 from Chapter 7 in S&B (pages 141)

Exercise 7.16

a) The original system and the reduced row echelon form are, respectively, $\begin{pmatrix} 1 & 2 & 1 & -1 & | & 1 \\ 3 & -1 & -1 & 2 & | & 3 \\ 0 & -1 & 1 & -1 & | & 1 \\ 2 & 3 & 3 & -3 & | & 3 \end{pmatrix}$

and $\begin{pmatrix} 1 & 0 & 0 & 3/11 & | & 12/11 \\ 0 & 1 & 0 & -1/11 & | & -4/11 \\ 0 & 0 & 1 & -12/11 & | & 7/11 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$

The variable z is free and the rest are basic. The solution is

$$w = \frac{12}{11} - \frac{3}{11}z$$

$$x = -\frac{4}{11} + \frac{1}{11}z$$

$$y = \frac{7}{11} + \frac{12}{11}z$$

b) The original system and the reduced row echelon form are, respectively

$$\begin{pmatrix} 1 & -1 & 3 & -1 & | & 0 \\ 1 & 4 & -1 & 1 & | & 3 \\ 3 & 7 & 1 & 1 & | & 6 \\ 3 & 2 & 5 & -1 & | & 3 \end{pmatrix} \text{and} \begin{pmatrix} 1 & 0 & 11/5 & -3/5 & | & 3/5 \\ 0 & 1 & -4/5 & 2/5 & | & 3/5 \\ 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$$

The variables y and z are free and the rest are basic. The solution is

$$w = \frac{3}{5} - \frac{11}{5}y + \frac{3}{5}z$$

$$x = \frac{3}{5} + \frac{4}{5}y - \frac{2}{5}z$$

c) The original system and the reduced row echelon form are, respectively $\begin{pmatrix} 1 & 2 & 3 & -1 & | & 1 \\ -1 & 1 & 2 & 3 & | & 2 \\ 3 & -1 & 1 & 2 & | & 2 \\ 2 & 3 & -1 & 1 & | & 1 \end{pmatrix}$

and $\begin{pmatrix} 1 & 0 & 0 & 0 & | & 17/65 \\ 0 & 1 & 0 & 0 & | & 7/65 \\ 0 & 0 & 1 & 0 & | & 22/65 \\ 0 & 0 & 0 & 1 & | & 32/65 \end{pmatrix}$

All variables are free and the rest are basic. The solution is $w = \frac{17}{65}, x = \frac{7}{65}, y = \frac{22}{65}, z = \frac{32}{65}$

d) The original system and the reduced row echelon form are, respectively $\begin{pmatrix} 1 & 1 & -1 & 2 & | & 3 \\ -1 & 2 & -2 & 4 & | & 6 \\ 3 & -3 & 3 & -6 & | & -9 \\ 2 & -2 & 2 & -4 & | & -6 \end{pmatrix}$ and $\begin{pmatrix} 1 & 1 & -1 & 2 & | & 3 \\ 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$

Variable w free and the rest are basic. The solution is

$w = 3 - x + y - 2z$

Exercise 7.18

The reduced row echelon form of the system is

$$\begin{pmatrix} 1 & 0 & | & 1 \\ 0 & 1 & | & 1 \\ 0 & 0 & | & 8 + a \end{pmatrix}$$

The last equation has solutions only when a=-8. In this case $x = y = 1$.

3

We write a system of equations that describes equilibrium in both markets simultaneously as follows:

$$A\boldsymbol{p} = \boldsymbol{b}$$

Where A is a 2×2 matrix of coefficients, $\boldsymbol{p}$ is a 2×1 vector of prices and $\boldsymbol{b}$ is a 2×1 vector of constants. The solution is given by

$$\boldsymbol{p} = A^{-1}\boldsymbol{b}$$

In order to express the system of equations that describes the markets for tea and coffee into the form $Ap = b$, we first need to obtain the equilibrium in each market, where $D_t = S_t$ and $D_c = S_c$. Then we obtain:

$$100 - 5p_t + 3p_c = -10 + 2p_t$$

$$120 - 8p_c + 2p_t = -10 + 5p_c$$

These can be rewritten more conveniently as

$$7p_t - 3p_c = 110$$

$$-2p_t + 13p_c = 130$$

In matrix form the above equations become

$$\begin{bmatrix} 7 & -3 \\ -2 & 13 \end{bmatrix} \begin{bmatrix} p_t \\ p_c \end{bmatrix} = \begin{bmatrix} 110 \\ 130 \end{bmatrix}$$

The solution is given by

$$\begin{aligned} \begin{bmatrix} p_t \\ p_c \end{bmatrix} &= \begin{bmatrix} 7 & -3 \\ -2 & 13 \end{bmatrix}^{-1} \begin{bmatrix} 110 \\ 130 \end{bmatrix} \\ &= \begin{bmatrix} 13/85 & 3/85 \\ 2/85 & 7/85 \end{bmatrix} \begin{bmatrix} 110 \\ 130 \end{bmatrix} \\ &= \begin{bmatrix} 21.41 \\ 13.29 \end{bmatrix} \end{aligned}$$