# Recitation #7 - (10/12/2018)

1. State the independence axiom. Show that if indifference curves in the Machina triangle are *not* parallel straight lines, then the independence axiom is violated.

   - *Independence axiom.* A preference relation $\succsim$ over the space of simple lotteries $\mathcal{L}$ satisfies the *independence axiom* if for all three lotteries $L, L', L'' \in \mathcal{L}$ and for all $\alpha \in [0,1]$, we have

     $$L \succsim L' \text{ if and only if } \alpha L + (1-\alpha) L'' \succsim \alpha L' + (1-\alpha) L''$$

     That is, if we mix each of two lotteries with a third one, then the preference ordering of the two resulting lotteries does not depend on (is independent of) the particular third lottery we use.

   - *Nonparallel indifference curves.* In order to show that when indifference curves are not parallel straight lines (as we can see in figure 1) the independence axiom is violated, we need to show that

     $$L \succsim L' \text{ does } \textit{not} \text{ imply } \alpha L + (1-\alpha) L'' \succsim \alpha L' + (1-\alpha) L''$$

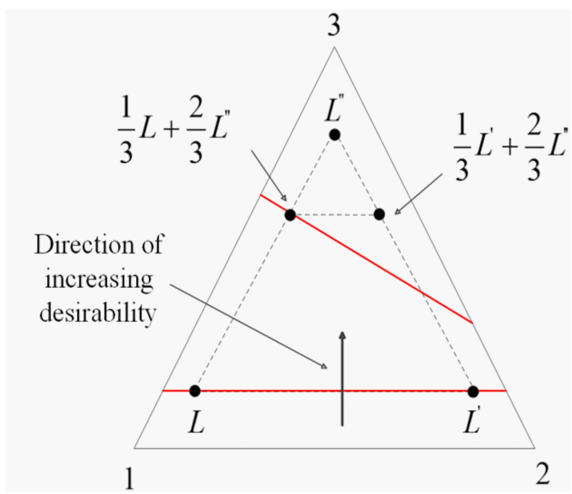


Figure 1. Nonparallel indifference curves.

This is easy to prove for the case in which the individual is indifferent between lotteries $L$ and $L'$, $L \sim L'$. That is, we must show

$$L \sim L' \text{ does } \textit{not} \text{ imply } \alpha L + (1-\alpha) L'' \sim \alpha L' + (1-\alpha) L''$$

Take the case in which $L \sim L'$, as lotteries $L$ and $L^{'}$ in Figure 1, which lie on the same indifference curve. As we can see, the mix of each of these two lotteries with a third lottery $L''$ leads to compound lotteries $\frac{1}{3}L + \frac{2}{3}L''$ and $\frac{1}{3}L' + \frac{2}{3}L''$, respectively. The possibility of indifference curves which are *not* parallel lines leads to situations like that in the figure, where

$$\frac{1}{3}L + \frac{2}{3}L'' \prec \frac{1}{3}L' + \frac{2}{3}L''$$

since the decision maker prefers the second compound lottery to the first. Hence, we cannot guarantee that $L \sim L^{'}$ implies

$$\alpha L + (1-\alpha)\, L'' \sim \alpha L' + (1-\alpha)\, L''$$

As a consequence, the independence axiom does not hold for indifference curves which are not parallel lines.

2. Consider an individual with preferences over lotteries that satisfy the independence axiom. Answer the following questions.

   (a) Show that the independence axiom implies convexity, i.e., for three different lotteries $L$, $L'$ and $L''$, if $L \succ L'$ and $L \succ L''$, then $L \succ \alpha L' + (1-\alpha)\, L''$.

   - From $L \succ L'$ we can apply the independence axiom, and obtain

     $$\alpha L + (1-\alpha)L \succ \alpha L' + (1-\alpha)L$$

     where note that we added $(1-\alpha)L$ on both sides of $L \succ L'$. Similarly, from $L \succ L''$ we can apply the independence axiom to obtain

     $$(1-\alpha)L + \alpha L' \succ (1-\alpha)L'' + \alpha L'$$

     where we added $\alpha L'$ on both sides of the strict preference relationship $L \succ L''$. By transitivity (from the two previous expressions), we have

     $$\alpha L + (1-\alpha)L \succ (1-\alpha)L'' + \alpha L'$$

     and rearranging

     $$L \succ \alpha L' + (1-\alpha)L''$$

     Intuitively, convex preference over lotteries means that if a decision maker prefers a lottery $L$ over either two lotteries, $L^{'}$ or $L^{''}$, then he must also prefer lottery $L$ over a convex combination of these two lotteries, $\alpha L^{'} + (1-\alpha)L^{''}$, i.e., the compound lottery of $L^{'}$ and $L^{''}$.

(b) Discuss why a decision maker whose preferences violate convexity can be offered a sequence of choices that lead him to a sure loss of money

- If a decision maker's preferences over lotteries violate convexity, then we must have that for three different lotteries $L$, $L'$ and $L''$, where $L \succsim L'$ and $L \succsim L''$, we obtain the opposite result than above; that is

$$\alpha L' + (1-\alpha)\, L'' \succ L$$

  Note that, if the decision maker initially owns the right to participate in lottery $L$, he will be willing to pay an amount $\$X$ in order to switch to the compound lottery $\alpha L' + (1-\alpha)\, L''$ given that $\alpha L' + (1-\alpha)\, L'' \succ L$. Now he owns the compound lottery $\alpha L' + (1-\alpha)\, L''$, and either lottery $L'$ or lottery $L''$ are realized. But we know that the decision maker prefers lottery $L$ to either of these lotteries since

$$L \succsim L' \;\; \text{and} \;\; L \succsim L''$$

  was an initial assumption of this decision maker's preferences over lotteries. Therefore, he would be willing to pay again $\$Y$ in order to obtain lottery $L$. Hence, the decision maker is exactly as at the starting point of this sequence of deals (lottery $L$) and has lost $\$X + \$Y$. We can then repeat the process again and again, and make this individual pay $\$X + \$Y$ dollars, keeping him exactly where he started! Essentially, this type of decision maker could be subject to a systematic explanation (the so-called Dutch books), being wiped out of the market place.