Homework #6 (Due on October 19th, 2018)

1. Consider an agent who exerts an effort level e, where $e \geq 0$, to generate output y that is subject to output shocks ε (e.g., weather conditions affecting the quality of harvest, machine breakdown causing product failure, etc.). Output then behaves as follows

$$y = ge + \varepsilon$$

where g denotes the agent's output efficiency, and shock ε follows a normal distribution, $N(0, \sigma^2)$, with mean of zero and variance $\sigma^2 > 0$. The agent earns a wage of $w = sy$, where $0 \leq s \leq 1$ represents his output shares (e.g., commission from sales of the products). In addition, the agent incurs a cost to exert effort e, given by

$$c(e) = \frac{1}{2}e^2$$

which is increasing and convex in his effort level e.

The agent's payoff comes from the utility of earning wage w minus his cost of effort, where

$$U = u(w) - c(e)$$

Specifically, his utility function follows the negative exponential form of

$$u(w) = 1 - \exp(-\eta w) \tag{1}$$

(a) What is the agent's Arrow-Pratt coefficient of absolute risk aversion, $r_A(w)$. How does it vary with his wage?

(b) Find the certainty equivalent of the agent.

(c) Find the agent's optimal effort e^*. How does it vary with his output share s, output efficiency g, risk aversion η, and output shocks σ?

2. Consider a similar setting to that in the previous exercise. However, we now analyze a worker exerting two levels of efforts, e_1 and e_2, where $0 \leq e_i \leq +\infty$ for every $i \in \{1, 2\}$. Intuitively, he may spend some hours a day ensambling products, some time presenting them to potential customers later on, and finally cleaning the store, or keeping records of all costs and receipts. For simplicity, assume that output y_1 (y_2) is only generated from effort e_1 (e_2), and let g_1 (g_2) denote the agent's efficiency at producing output 1

(2, respectively). Each output behaves similarly as in the previous exercise, that is,

$$y_1 = g_1 e_1 + \varepsilon_1$$
$$y_2 = g_2 e_2 + \varepsilon_2$$

Random shocks ε_1 and ε_2, which affect outputs 1 and 2 respectively, follow a bivariate normal distribution $N(\mathbf{0}, \Sigma)$, with expectation 0 for both outputs and variance-covariance matrix of

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}$$

The agent earns a wage of $w = s_1 y_1 + s_2 y_2$, where s_1 and s_2 represent his output shares, with $0 \leq s_i \leq 1$ for $i \in \{1, 2\}$. Cost is increasing and convex in both effort levels, where

$$c(e_1, e_2) = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2$$

such that his payoff function now becomes

$$U = u(w) - c(e_1, e_2)$$

and his utility function $u(w)$ is the same as that in part (a).

(a) Find the certainty equivalent payment of the agent.

(b) How is the agent's certainty equivalent affected when he becomes more risk averse?

(c) How is the agent's certainty equivalent affected when output shocks become more volatile?

(d) How is the agent's certainty equivalent affected when output shocks become more correlated?