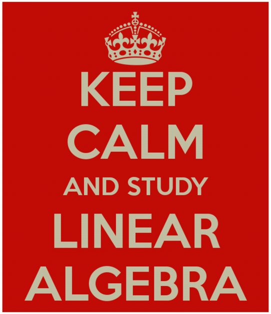
# Introduction to Linear Algebra

Chapter 6 -S&B



- Some of the most frequently studied economic models are linear. Examples?
- Linear equations are

$$x_1 + 2x_2 = 3 \text{ and } 2x_1 + 3x_2 = 8$$

- In general

$$a_1x_1 + a_2x_2 + ... + a_nx_n = b$$

- Let us identify: *Parameters* and *Variables*
- The variable appears only to the first power
- Advantage: we can often calculate exact solutions to the equations
    - nonlinear systems?
- Advantage: precises relationship between the solution of the linear system and various parameters

# Linear Models

- We can learn about the behavior of a NL system by studying suitably chosen linear approximations to the system.
- Example: *Tax Benefit of Charitable Contributions*
    - A company earns before-tax profits of $\$100,000$. It has agreed to contribute 10 percent of its after-tax profits to the Red Cross Relief Fund. It must pay a state tax of 5 percent of its protits (after the Red Cross donation) and a federal tax of 40 percent of its profits (after the donation and state taxes are paid). How much does the company pay in state taxes, federal taxes, and Red Cross donation?

# Example Linear Model of Production

- Assume that our economy has $n+1$ goods.
- Each of goods 1 through n is produced by one production process.
- There is also one commodity, labor (good 0).
- A production process is simply a list of amounts of goods: so much of good 1. so much of good 2, and so on.
    - These quantities are the amounts of input needed to produce one unit of the process's output.
- **Constant returns to scale**. The production of 2, 3, or $k$ cars requires 2, 3, or $k$ times the amounts of inputs required for the production of 1 car.

# Example Linear Model of Production

- Consider the economy of an organic farm which produces two goods: corn and fertilizer. Corn is produced using corn
- (to plant) and fertilizer.
- Suppose that the production of 1 ton of corn requires as inputs 0.1 ton of corn and 0.8 ton of fertilizer. The production of 1 ton of fertilizer requires no fertilizer and 0.5 ton of corn

$$\begin{aligned} a &: \textit{corn} \text{ and } (0.1, 0.8) \\ b &: \textit{fertilizer} \text{ and } (0.5, 0.) \end{aligned}$$

# Example Linear Model of Production

- What can be produced for consumption? Is there any way of running both processes so as to leave some corn and some fertilizer for individual consumption? If so, what combinations of corn end fertilizer for consumption are feasible?

# Example Linear Model of Production

- *In general terms: input-output coefficients*

$$\{a_{0j}, a_{1j}, a_{2j}, ..., a_{nj}\}$$

- $a_{ij}$ denotes the input of good $i$ needed to output one unit of good $j$
- The production of $x_j$ units of good $j$ requires $a_{0j}x_j$ units of good 0, $a_{1j}x_j$ units of good 1, and so on.
- Denote by $c_i$ the consumer demand for good $i$. This demand is given *exogenously* (not solved for in the model).
- Let $c_0$ be the consumer's supply of labor. Since good 0 (labor) is supplied by consumers rather than demanded by consumers
- $c_0$ is a negative number. An n-tuple $(c_0, c_1, c_2, .., c_n)$ is the admissible n-tuple of consumes demands.

# Example Linear Model of Production

- The law of supply and demand: output produced must be used in production or in consumption.
- Let $x_j$ denote the amount of output produced by process $j$. If process $j$ produces $x_j$ units of output, it will need $a_{ij}x_j$, units of good $i$.

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n + c_i \quad \textit{demand for good } i$$

$$x_i = a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n + c_i$$

- Let us obtain the **Open Leontief System**

# Example Markov Models of Employment

- Aggregate unemployment rate do not tell the whole story of unemployment.
- is most unemployment due to a few people who are unemployed for long periods of time, or is it due to many people, each of whom is only briefly unemployed?

# Example Markov Models of Employment

*If an individual is not employed in a given week.* $p$ is the probability of finding a job and $(1-p)$ is the probability of remaining unemployed


*If an individual is employed in a given week.* $q$ is the probability of remaining employed and $(1-q)$ is the probability of becoming unemployed.

# Example Markov Models of Employment

- **Transition probabilities**: $p$, $q$, $(1-p)$ and $(1-q)$
- The chances of finding a job are independent of how many weeks the job seeker has been unemployed and that the chances of leaving a job are also independent of the number of weeks worked.

# Example Markov Models of Employment

- The transition probabilities can lead to a description of the pattern of unemployment over time.
- Suppose that there are $x$ males of working age who are currently employed and $y$ who are currently unemployed. How will these numbers change next week?

$$qx + py \text{ average number employed}$$
$$(1-q)x + (1-p)y \text{ average number unemployed}$$

$$\begin{aligned} x_{t+1} &= qx_t + py_t \\ y_{t+1} &= (1-q)x_t + (1-p)y_t \end{aligned}$$

- Linear system of difference equations

# Example IS-LM Analysis

- Keynes' classic work "*General Theory of Employment, Interest, and Money*"
- Consider an economy with no imports, exports, or other leakages. In this economy, the value of total production equal total spending, which in turns equals total national income, $Y$.

$$Y = C + I + G$$

- On the consumer side,

$$C = bY$$

- where $b \in [0, 1]$ is called the *marginal propensity to consume* and $s = 1 - b$ is called the *marginal propensity to save*

# Example IS-LM Analysis

- On the firms' side:
$$I = I^0 - ar$$
- where $a$ is the *marginal efficiency of capital.* Then putting all together we get the IS schedule
$$Y = bY + \left(I^0 - ar\right) + G$$
- wich can be expressed as follows
$$sY + ar = I^0 + G \tag{1}$$
- This equation describes the real life side of the economy, since it summarizes consumption, investment and saving decisions

# Example IS-LM Analysis

- Let's analyze the LM equation. LM it is determined by the monetary market eq. condition $M_S = M_d$. MS is determined outside the system.
- $Md$ has two components: the transaction demand, $M_{dt}$, and the speculative demand $M_{ds}$

$$M_{dt} = mY$$

- as national income rises, $Y$, so does the demand.

$$M_{ds} = M^o - hr$$

- it varies inversely with the interest rate. Hence, since $M_S = M_d$

$$M_S = mY + M^o - hr$$

- or

$$mY - hr = M_S - M^o \quad (2)$$

- where the equilibrium income, $Y$, and interest rate, $r$, are