Recitation #6 (09/28/2018) - Production Theory

1. Consider the following profit function that has been obtained from a technology that uses a single input, z:

$$\pi(p, w) = p^2 w^\alpha$$

where p is the output price, w is the input price and α is a parameter value.

   (a) Check if the profit function satisfies homogeneity of degree one jointly in both p and w. In particular, determine for which values of α this property is satisfied.

   (b) Assuming the value of α for which the profit function satisfies homogeneity of degree one, check if the profit function $\pi(p, w)$ satisfies the following properties: (1) non-decreasing in output price p, (2) non-increasing in input prices w, and (3) convex in prices p and w.

   (c) Calculate the supply function of the firm, $q(p, w)$, and its demand for inputs, $z(p, w)$.

2. The profit function, $\pi(p)$, is defined as

$$\pi(p) = \max \{p \cdot y \mid y \in Y\}$$

or alternatively, $\pi(p) \geqslant p \cdot y$ for every feasible production plan $y \in Y$.

   (a) Show that the profit function $\pi(p)$ is convex in prices.

   (b) Prove that Hotelling's lemma holds for this profit function.

   (c) Show that, if the output function $y(p)$ is differentiable at $\overline{p}$, then $D_p y(\overline{p})$ is a symmetric and positive semidefinite matrix.