Recitation #5 (09/21/2018)

1. Consider a representative consumer with utility function $u(q_1, q_2) = \ln q_1 + q_2$,where q_1 denotes gallons of gas and q_2 is a numeraire representing all other goods. The price of q_2 is therefore normalized to one, $p_2 = 1$, while the price of gas is $p_1(1 + t)$, where $t \in [0, 1]$ represents a specific tax per gallon of gas. For simplicity, assume that the consumer's income is $m > 0$.

   (a) Find the Walrasian demand for q_1 and q_2, denoting them as q_1^W and q_2^W, and distinguish the case in which $m > 1$ and that when $m \leq 1$.

   (b) Find the associated indirect utility function, $v\left(q_1^W, q_2^W\right)$. After months of lobbying from consumers' associations, the government is considering implementing either of the following policies: (1) reduce the tax on gas, from t to $t' = t - \alpha$; or (2) maintain the tax at t but give a subsidy of S dollars to the consumer equal to the tax revenue collected by the tax on gas.

   (c) Let us first consider that the consumer's income satisfies $m > 1$, i.e., the consumer is relatively rich. Find the consumer's indirect utility function if the government implements the first policy, $v^I\left(q_1^W, q_2^W\right)$, and if the government implements the second policy, $v^{II}\left(q_1^W, q_2^W\right)$. Under which conditions does the consumer prefer the first policy?

   (d) Let us now consider that the consumer's income satisfies $m \leq 1$, i.e., the consumer is relatively poor. Find the consumer's indirect utility function if the government implements the first policy, $v^I\left(q_1^W, q_2^W\right)$, and if the government implements the second policy, $v^{II}\left(q_1^W, q_2^W\right)$. Under which conditions does the consumer prefer the first policy?

2. Consider a Cobb-Douglas production function $f : \mathbb{R}_+^2 \longrightarrow \mathbb{R}_+$, given by $f(z) = 2^{3/4} z_1^{1/4} z_2^{1/4}$, where $z_1 \geqslant 0$ and $z_2 \geqslant 0$ denote inputs in the production process.

   (a) Check if the production function has nonincreasing, nondecreasing, or constant returns to scale.

   (b) Let $w \in \mathbb{R}_{++}^2$ denote the vector of input prices and $p > 0$ the output price. Determine for each output level $q \geqslant 0$ the cost function $c(w, q)$ and the conditional factor demand $z(w, q)$.

   (c) Verify Shephard's lemma.

   (d) Determine the profit function $\pi(p, w)$.