Recitation #4 (09/14/2018)- Solution

1. An individual consumes only goods 1 and 2, and his indirect utility function, $v(p_1, p_2, w)$, is given by the following expression

$$v(p_1, p_2, w) = \frac{w}{p_1 + \alpha p_2} \quad \text{where } \alpha > 0, \text{ and } p_1,\ p_2,\ w > 0$$

   (a) Find this individual's Walrasian demand for good 1, $x_1(p, w)$, and for good 2, $x_2(p, w)$, where p denotes the price vector $p \equiv (p_1, p_2)$. [*Hint*: Use an equivalence in order to go from indirect utility function to Walrasian demand in only one step.] Then, find the ratio

$$\frac{x_2(p, w)}{x_1(p, w)}$$

   Explain the intuition behind your result.

   - In order to find the Walrasian demands, we need to use Roy's identity, $x_i(p, w) = -\frac{\frac{\partial v(p,w)}{\partial p_i}}{\frac{\partial v(p,w)}{\partial w}}$. In particular, for good 1 we find a Walrasian demand of

$$x_1(p, w) = -\frac{\frac{\partial v(p,w)}{\partial p_1}}{\frac{\partial v(p,w)}{\partial w}} = -\frac{\frac{-w}{(p_1+\alpha p_2)^2}}{\frac{1}{p_1+\alpha p_2}} = \frac{w}{p_1 + \alpha p_2},$$

     and, similarly, good 2's Walrasian demand is

$$x_2(p, w) = -\frac{\frac{\partial v(p,w)}{\partial p_2}}{\frac{\partial v(p,w)}{\partial w}} = -\frac{\frac{-w\alpha}{(p_1+\alpha p_2)^2}}{\frac{1}{p_1+\alpha p_2}} = \alpha \frac{w}{p_1 + \alpha p_2}.$$

     Hence, the ratio

$$\frac{x_2(p, w)}{x_1(p, w)} = \frac{\alpha \frac{w}{p_1+\alpha p_2}}{\frac{w}{p_1+\alpha p_2}} = \alpha$$

   - That is, the demand of good 2 is a constant proportion, α, of the demand of good 1. This constant proportion holds for any wealth level, and any price vector. Since goods are consumed in constant proportions, these two goods are perfect complements (Leontief indifference curves).

   (b) Find this individual's Hicksian demand for good 1, $h_1(p, u^0)$, and good 2, $h_2(p, u^0)$. [*Hint*: Use equivalences in this part of the exercise as well: one to go from indirect utility function to expenditure function, and another to go from expenditure function to hicksian demand.] Then, find the ratio

$$\frac{h_2(p, u^0)}{h_1(p, u^0)}$$

Explain the intuition behind your result.

- We know that we can find Hicksian demands from the expenditure function by using Shephard's lemma. In order to find the expenditure function, we first use the following properties,

$$w \equiv e(p, u^0) \quad \text{and} \quad v(p, w) \equiv u^0$$

Using them in the expression of the indirect utility $v(p, w) = \frac{w}{p_1 + \alpha p_2}$, we have

$$u^0 = \frac{e(p, u^0)}{p_1 + \alpha p_2}$$

and solving for the expenditure function, $e(p, u^0)$, we obtain

$$e(p, u^0) = u^0(p_1 + \alpha p_2)$$

Now that we have the expenditure function, we can use Shephard's lemma in order to find the Hicksian demands,

$$h_1(p, u^0) = \frac{\partial e(p, u^0)}{\partial p_1} = \frac{\partial [u^0(p_1 + \alpha p_2)]}{\partial p_1} = u^0, \text{ and}$$

$$h_2(p, u^0) = \frac{\partial e(p, u^0)}{\partial p_2} = \frac{\partial [u^0(p_1 + \alpha p_2)]}{\partial p_2} = \alpha u^0$$

Hence, the ratio

$$\frac{h_2(p, u^0)}{h_1(p, u^0)} = \frac{\alpha u^0}{u^0} = \alpha$$

This result (goods are consumed in constant proportions) just confirms our previous result of part (a).

(c) Using the Walrasian and Hicksian demands you found in parts (a) and (b), find the Slustky equation for goods 1 and 2. Explain your result, and connect it with your intuitions on parts (a) and (b).

- The Slutsky equation can be written as follows

$$\frac{\partial x_i(p, w)}{\partial p_j} = \frac{\partial h_i(p, u^0)}{\partial p_j} - x_j(p, w)\frac{\partial x_i(p, w)}{\partial w}$$

But, from the Hicksian demands found in the previous part of the exercise,

$h_1(p, u^0) = u^0$ and $h_2(p, u^0) = \alpha u^0$, we have that

$$\frac{\partial h_i(p, u^0)}{\partial p_i} = \frac{\partial h_i(p, u^0)}{\partial p_j} = 0 \text{ for all } i, j = 1, 2$$

- Therefore, the Slutsky equation becomes

$$\frac{\partial x_i(p, w)}{\partial p_j} = -x_j(p, w)\frac{\partial x_i(p, w)}{\partial w} \quad \text{for all } i, j = 1, 2$$

This result should come at no surprise: parts (a) and (b) of the exercise emphasize that these goods are consumed in constant proportions (they are perfect complements in consumption) which implies that there is no substitution among them (i.e., the total effect contains no substitution effect, but only an income effect).

(d) Let us now assume that the initial price of good 1 doubles, the price of good 2 is cut in half, and wealth is kept constant. That is, denoting by $p^0 \equiv (p_1^0, p_2^0)$ the vector of initial prices and $p^1 \equiv (p_1^1, p_2^1)$ the vector of final prices, we have that

$$p_1^1 = 2p_1^0 \text{ for good 1, and } p_2^1 = \frac{1}{2}p_2^0 \text{ for good 2.}$$

(i) Find the compensating variation (CV) due to the price change. Explain intuitively what CV measures.

- The compensating variation, CV, is the monetary amount that, *after* the price change, must be given to the individual in order to maintain him at the same indifference curve he was *before* the price change, i.e.,

$$CV = e(p^1, u^0) - e(p^0, u^0)$$

Using the expenditure function that we found in part (b) of the exercise,

$$\begin{aligned} CV &= e(p^1, u^0) - e(p^0, u^0) \\ &= u^0(p_1^1 + \alpha p_2^1) - u^0(p_1^0 + \alpha p_2^0) \\ &= u^0\left(2p_1^0 + \alpha\frac{1}{2}p_2^0\right) - u^0(p_1^0 + \alpha p_2^0) = u^0\left(p_1^0 - \alpha\frac{1}{2}p_2^0\right) \end{aligned}$$

(ii) Find the equivalent variation (EV) due to the price change. Explain intuitively what EV measures.

- The equivalent variation, EV, is the monetary amount that, *before* the price change, must be given to the individual in order to make him reach the same indifference curve he would reach *after* the price change, i.e.,

$$EV = e(p^1, u^1) - e(p^0, u^1)$$

- Using the expenditure function that we found in part (b), we can find the equivalent variation

$$\begin{aligned} EV &= e(p^1, u^1) - e(p^0, u^1) \\ &= u^1(p_1^1 + \alpha p_2^1) - u^1(p_1^0 + \alpha p_2^0) \\ &= u^1\left(2p_1^0 + \alpha\frac{1}{2}p_2^0\right) - u^1(p_1^0 + \alpha p_2^0) = u^1\left(p_1^0 - \alpha\frac{1}{2}p_2^0\right) \end{aligned}$$