

## EconS527 – Practice Exercises September 24th, 2018

### Question #1

Firms *A* and *B* can choose to adopt a new technology (*N*) or to adhere to their old technology (*O*). Formally, firms' action sets are:  $t_A \in \{N, O\}$  and  $t_B \in \{N, O\}$ . The table below exhibits the profit made by each firm under different technology choices.

		<i>Firm B</i>	
		<i>New Technology</i>	<i>Old Technology</i>
<i>Firm A</i>	<i>New</i>	(200,0)	(0,200)
	<i>Old</i>	(50,100)	(100,50)

- a) Write down the best-response functions of firms *A* and *B*,  $t_A = R_A(t_B)$  and  $t_B = R_B(t_A)$
- b) Draw the tree of a two-stage extensive-form game in which firm *A* chooses its technology  $t_A$  in stage I, and Firm *B* chooses its  $t_B$  in stage II (after observing the choice made by firm *A*). Make sure that you indicate firms' profits at the termination points on the tree. Solve for the *subgame-perfect equilibrium* of this game. Provide a short proof or an explanation justifying your answer.
- c) Draw the tree of a two-stage extensive-form game in which firm *B* chooses its technology  $t_B$  in stage I and Firm *A* chooses its  $t_A$  in stage II (after observing the choice made by firm *B*). Solve for the *subgame-perfect equilibrium* of this game. Provide a short proof or an explanation justifying your answer.
- d) Compare the equilibrium firms' profit levels of the games played in (b) and in (c). Conclude under which game firm *A* earns a higher profit. Briefly explain your answer.

### Question #2

Consider the two-player game in the following payoff matrix, where  $a_i \neq b_i \neq c_i \neq d_i$  for every player  $i=\{1,2\}$ . If the game has a unique pure strategy Nash equilibrium (psNE), show that it must be the unique strategy profile that survives IDSDS.

		Player 2	
		$s_2^A$	$s_2^B$
Player 1	$s_1^A$	$a_1, a_2$	$c_1, c_2$
	$s_1^B$	$b_1, b_2$	$d_1, d_2$

**Question #3**

Consider the following 2x2 game

		Player 2	
		L	R
Player 1	L	(9,9)	(0,8)
	R	(8,0)	(7,7)

- a) By inspection, what are the pure strategy Nash equilibria?
- b) Find the additional mixed strategy equilibrium by using the fact that if a player is willing to mix between two or more strategies, she will be indifferent between them in equilibrium.
- c) Draw the best-response correspondences. Where do they intersect?

**Question #4**

Consider the following incomplete-information game: Players 1 and 2 simultaneously and independently choose "Work" or "Shirk", but there is a 0.5 probability that the players are playing the game on the left matrix, and a 0.5 probability that the play on the right matrix. Player 1 knows which game is being played, but player 2 does not. Intuitively, the left-hand (right-hand) matrix represents a setting in which the benefit of having both individuals working is relatively low (high, respectively).

		Player 2				Player 2	
		Work	Shirk			Work	Shirk
Player 1	Work	5, 5	-3, 6	Player 1	Work	10, 10	-3, 6
	Shirk	6, -3	-1, -1		Shirk	6, -3	-1, -1

- a) Describe the strategy space for each player.
- b) Depict the Bayesian-normal form representation of the game.
- c) Find the Bayesian Nash equilibrium (BNE) of this game, both in pure and mixed strategies.