Homework #4 - EconS 527 (Due on 10/08)

1. Consider the following simultaneous-move game between player 1 (in rows) and player 2 (in columns).

		Player 2		
		L	M	R
	U	$4, 2$	$2, 0$	$0, 3$
Player 1	C	$5, 1$	$3, 2$	$\delta, 4$
	D	$5, 2$	$6, 2$	α, β

   We are informed that player 1 finds that strategy C weakly dominates U. In addition, we are told that strategy profile (D, R) is a Nash equilibrium but (D, M) is not. Using this information, answer the following questions.

   Before we start, let us define $u_i(s_i, s_j)$ to be the utility function of player i when player i deploys strategy $s_i \in S_i$ and his counterpart, player j, deploys strategy $s_j \in S_j$, where $i, j = \{1, 2\}$. In this context, $S_1 = \{U, C, D\}$ and $S_2 = \{L, M, R\}$.

   (a) Does player 2 have a strictly dominated strategy?

   (b) Is strategy profile (D, R) the unique Nash equilibrium of this game?

   (c) Does player 1 have a strictly dominant strategy?

2. Consider a sequential-move bargaining game between Player 1 (proposer) and Player 2 (responder). Player 1 makes a take-it-or-leave-it offer to Player 2, specifying an amount $s = \{0, \frac{1}{2}v, v\}$ out of an initial surplus v, i.e., no share of the pie, half of the pie, or all of the pie. If Player 2 accepts such a distribution Player 2 receives the offer s, while Player 1 keeps the remaining surplus $v - s$. If Player 2 rejects, both players get a zero payoff.

   (a) Describe the strategy space for every player.

   (b) Provide the normal-form representation of this bargaining game.

   (c) Does any player have strictly dominated pure strategies?

   (d) Does any player have strictly dominated mixed strategies?

3. Consider a Cournot duopoly with linear inverse demand curve $p(q) = a - q$, where q denotes aggregate output. Both firms have a common constant marginal cost $c > 0$, and where $a > c$. Assume that firms do an equity swap of γ, i.e., each firm i receives a share $0 < \gamma \leq \frac{1}{2}$ in firm j's profits, where $j \neq i$.

(a) Find the Cournot equilibrium output, (q_1^C, q_2^C).

(b) Evaluate equilibrium output q_i^C at $\gamma = 0$ and $\gamma = \frac{1}{2}$. Interpret.

(c) Determine if q_i^C increases or decreases in γ.

(d) Find equilibrium profits, π^C, and determine whether they increase or decrease in γ.

4. Consider a Cournot duopoly where firms sell their production in a competitive market (e.g., an international market where the duopolists' sales represent a small share of total sales) at prices $p_1 = \$2$ and $p_2 = \$3$. Both firms face a concave supply function, $q_1 = 13x_1 - 0.2x_1^2$ for firm 1 and $q_2 = 12x_2 - 0.1x_2^2$ for firm 2, where x_1 denote the amount of input (e.g., labor) that firm 1 hires; and similarly for x_2, which indicates the amount of input that firm 2 hires. Assume that the cost of hiring x_i units of input for firm i when its rival hires x_j units of that input is $C(x_i, x_j) = [2 + 0.1(x_i + \beta x_j)]\, x_i$ where β represents the cost externality that firm i suffers from every unit of input firm j hires. Specifically, when $\beta = 0$, the above cost function collapses to $C(x_i, x_j) = [2 + 0.1x_i]\, x_i$, thus being independent on firm j's hiring decisions. In contrast, when $\beta > 0$, firm j's hiring decisions increase firm i's cost. For instance, high skill workers may become more scarce, and thus firm i needs to offer them a higher salary to attract them to work for firm i.

   (a) Write down firm 1's profit-maximization problem. Find this firm's best response function. Evaluate it at $\beta = 0$ and at $\beta > 0$. Interpret. Repeat your analysis for firm 2.

   (b) Determine the equilibrium values of firms' hiring decisions, x_1 and x_2.

   (c) How are the equilibrium results from part (b) affected by a marginal increase in the cost externality parameter β?