EconS 527- Homework #1 (Due on September 3rd, 2018)

1. Consider the following three bundles:

$$(x_1,x_2) = (2,2)$$
$$(y_1,y_2) = (4,1)$$
$$(z_1,z_2) = (5,2)$$

Is transitivity satisfied? Discuss your answer [Hint: consider the example about indistinguishable alternatives discussed during class]

Solution

Note that $(y_1,y_2)\succsim(x_1,x_2)$ *since the difference in their first component is greater to one unit,* $x_1\geq y_1-1$ *(i.e.,* $2 \ngeq 4-1$*). Additionally,* $(y_1,y_2) \succsim (z_1,z_2)$ *is also satisfied since* $y_1 \geq z_1 -1$*.Finally,* $(z_1,z_2\succsim x_1,x_2)$ *since the difference between* z_1 *and* x_1 *is larger than one unit,* $z_1\geq x_1 - 1$ *(i.e.,* $5 \geq 2 - 1$*). Hence, this preference relation satisfies Transitivity*

2. Give an example (different than the one discussed in class) to support that changes in preferences violates transitivity.

Solution

During class we discussed the example of a smoker.

3. Discuss whether the following functions are monotone or strongly monotone:

$$v(y,z) = \min\{3y,2z\}$$

$$v(y,z) = 5y + 3z$$

In addition, discuss if the above functions satisfy convexity or/and strict convexity.

Solution

v(y,z) = min{3y,2z} is monotone but not strongly monotone. The optimal solution is at $3y = 2z$. So $v(y,z)$ only increases when both y and z increase. Sole increase of y or z does not affect $v(y,z)$. It satisfies convexity but not strict convexity. Proof is as follows.

Proof:

$v(y,z) = \ min\{3y,2z\}$ is convex but not strictly convex. Connecting any two points $\{y_1,z_1\}$ and $\{y_2,z_2\}$ of function v, for all $t \in (0,1)$, we have

$$v(t(y_1+y_2),(1-t)(z_1+z_2)) \le tv(y_1,z_1)+(1-t)v(y_2,z_2)$$

The left-hand side of the inequality is not strongly smaller than the right-hand side. According to the definition of convexity, we thus know function v is convex, but not strongly convex.

v(y,z) = 5y + 3z are strongly monotone because $\frac{\partial v(y,z)}{\partial y}=5>0$ and $\frac{\partial v(y,z)}{\partial z}=3>0$.

The function satisfies convexity but it also satisfies concavity.

Proof:

For any two points (x_1,y_1) and (x_2,y_2) on the function, we have

$$\begin{aligned} v(ay_1+(1-a)y_2, az_1+(1-a)z_2) &= 5(ay_1+(1-a)y_2)+3(az_1+(1-a)z_2) \\ &= 5ay_1-5ay_2+5y_2+3az_1-3az_2+3z_2 \\ &= a(5y_1+3z_1)+(1-a)(5y_2+3z_2) = av(y_1,z_1)+(1-a)v(y_2,z_2) \\ &\le av(y_1,z_1)+(1-a)v(y_2,z_2) \end{aligned}$$

where $a\in(0,1)$. So this function is both convex and concave (by flipping the sign of $\le$) but not strong convex.

4. Explain monotonicity and strong monotonicity in preference relations, and compare them. Provide an example where a bundle x is (strictly) preferred to bundle y when preferences satisfy strong monotonicity, but x is not necessarily preferred to y under monotonicity.
Solution

- *Monotonicity* states that increasing the amount of some commodities cannot hurt, and increasing the amount of all commodities is strictly preferred. Formally, if we take bundle $y\in R^L$ and weakly increase all k components, so that we generate a new bundle $x\in R^L$ satisfying $x_k \ge y_k$ for all k, then an individual with monotonic preferences would prefer the newly created bundle to the original bundle, i.e., $x \succsim y$. (Note that this implies that at least one component of the bundle has been strictly increased while the remaining components can be left unaffected.) In addition, if we strictly increase the amount of all components in bundle y, this individual would strictly prefer the new bundle, i.e., if $x_k \ge y_k$ for all k, then $x \succ y$.
- *Strong monotonicity*. On the other hand, strong monotonicity states that the consumer is strictly better off with additional amounts of any commodity. That is, if we strictly increase the amount of at least one commodity, the consumer strictly prefers the newly created bundle x to his original bundle y. That is, if $x_k \ge y_k$ for all good k and $x\neq y$, then $x \succ y$. (Note that this implies that $x_j \ge y_j$ for at least one commodity j, since otherwise both bundles would coincide.)
- *Comparison*. Then, a consumer's preference relation can satisfy monotonicity (if additional amounts of one of his commodity do not harm his utility), but does not need to satisfy

strong monotonicity (since for that to occur, he would need to become strictly better off as a consequence of the additional amounts in one of his commodities). However, if a consumer's preferences satisfy strong monotonicity, they must also satisfy monotonicity. That is why strong monotonicity is a more restrictive ("stronger") assumption on preferences than monotonicity.

- **Example:** Consider bundles x=(1,2) and y=(1,1). If preferences satisfy strong monotonicity, $x \succ y$ since the second component in bundle x is higher than the corresponding component in y, i.e., $x_j \geq y_j$ for some good j. However, if preferences only satisfy monotonicity, we cannot state that $x \succ y$ (strictly), since $x_k > y_k$ does not hold for all k commodities.

5. Discuss the main differences between the Choice Based Approach and the Preference-Based Approach. Provide an example that highlights such differences.

Solution

Both approaches have their own advantages. For instance, the preference-based approach is based on unobservables (the individual's preferences), while the choice-based approach is based on observables (the actual choices made by the individual decision-maker). On the other hand, the preference-based approach is more tractable than the choice-based approach, especially when the set of alternatives X contains many elements (which is usually the case in individual decision-making problems). Example: Choosing transportation when coming to class.