Homework #4 (Due on September 21st, 2018)

1. Assume that you have friend, Leo, who is now retired and lives on a fixed income $w > 0$ which does not adjust to inflation. His expenditure function is

$$e(p_1, p_2, u) = (p_1 + p_2)u,$$

where $p_1, p_2 > 0$ denote initial prices. Suppose that prices of goods 1 and 2 increase to p_1' and p_2', respectively.

   (a) You want to give him a monetary gift so that he will not be affected by the above price increase. How much money should you give him? That is, find his compensating variation (CV).

   (b) Now find his equivalent variation (EV) from the price change, i.e., the change in income needed at initial prices p_1 and p_2 that would have the same effect on utility as would the change in prices, p_1' and p_2'.

   (c) Which is larger in this case, CV or EV?

   (d) Find his Walrasian demand for each good.

   (e) Find his utility function. What is this type of utility function called?

2. Show that the compensating and the equivalent variation coincide when the utility function is quasilinear with respect to the first good (and we fix $p_1 = 1$). [*Hint*: Use the definitions of the compensating and equivalent variations in terms of the expenditure function (not the hicksian demand). In addition, recall that if $u(x)$ is quasilinear with respect to good 1, then we can express it as

$$u(x) = x_1 + \phi(x_{-1}),$$

where x_{-1} represents all the remaining goods, $l = 2, 3, ..., L$.]

3. Chelsea loves chocolate (x) and books (y), and her utility from consuming these two goods can be represented by a with quasilinear utility function $u(x, y) = \rho\sqrt{x} + \tau y$, where $\rho, \tau > 0$.

   (a) Find the Walrassian demand of the individual.

   (b) Find the Hicksian demand for goods 1 and 2.

   (c) Assume that Chelsea's wealth is $w = \$500$, and prices are $p_1 = p_2 = \$15$. For simplicity, consider parameters $\rho = 1, \tau = \frac{1}{2}$. Find the AV, CV and EV.

4. Consider an individual with utility function $u(q_1, q_2) = q_1^2 + q_2 - 1$, where q_1 (q_2) denotes the units of good 1 (good 2, respectively) that this individual consumes. His income level is denoted by $w \in \mathbb{R}_+$, and prices are both strictly positive, i.e., $\mathbf{p} = (p_1, p_2) \in \mathbb{R}^2_{++}$.

   (a) Determine this individual's Walrasian demand, and his associated indirect utility function.

   (b) Determine this individual's Hicksian demand, $h_1(\mathbf{p}, u)$ and $h_2(\mathbf{p}, u)$, and his associated expenditure function, $e(\mathbf{p}, u)$.

Consider now that this individual's income level is $w = 6$, and the initial vector of market prices is $\mathbf{p^0} = (4, 3)$. If both prices increase by 50%, determine:

1. (c) The compensating variation of this price increase. Interpret.

   (d) The change in consumer surplus associated to this price increase. Interpret.