EconS 527- Homework #3 Answer Key

1. The certainty equivalent of a lottery is the amount of money you would have to be given with certainty to be just as well-off with that lottery. Suppose that you von Neumann-Morgenstern utility function over lotteries that give you an amount of x if Event 1 happens and y if Even 1 does not happen is $U(x, y, \pi) = \pi\sqrt{x} + (1-\pi)\sqrt{y}$, where π is the probability that Event 1 happens and $1-\pi$ is the probability that Event 1 does not happen.

a) $.5 \times \sqrt{10{,}000} + 5 \times \sqrt{100} = .5 \times 100 + .5 \times 10 = 55$

b) If you were sure to receive \$4,900, then you receive \$4,900 in both events. $\pi \times \sqrt{4{,}900} + (1-\pi) \times \sqrt{4{,}900} = 70$, where π can be any probability.

c) $(.5x^{\frac{1}{2}} + .5y^{\frac{1}{2}})^2$

d) Using the equation in part c) $(.5x^{\frac{1}{2}} + .5y^{\frac{1}{2}})^2 = (.5(10{,}000)^{\frac{1}{2}} + .5(100)^{\frac{1}{2}})^2 = 3{,}025$

2.

11.6.a) Note that

$$E[u(R)] = \int_{-\infty}^{\infty} u(s) \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{s-\mu}{\sigma}\right)^2\right\} ds$$
$$= \phi(\mu, \sigma^2)$$

11.6.b) Normalize $u(\cdot)$ such that $u(\mu) = 0$. Differentiating, we have

$$\frac{\partial E[u(R)]}{\partial \mu} = \frac{1}{\sigma^2} \int_{-\infty}^{\infty} u(s)(s-\mu) f(s) ds > 0$$

since the terms $[u(s)(s-\mu)]$ and $f(s)$ are positive for all s.

11.6.c) Now we have

$$\frac{\partial E[u(R)]}{\partial \sigma^2} = \frac{1}{\sigma^3} \int_{-\infty}^{\infty} u(s)((s-\mu)^2 - \sigma^2) f(s) ds$$

$$<\frac{1}{\sigma^3}\int_{-\infty}^{\infty} u'(\mu)(s-\mu)((s-\mu)^2-\sigma^2)f(s)ds$$

$$=\frac{u'(\mu)}{\sigma^3}\{\int_{-\infty}^{\infty}(s-\mu)^3 f(s)ds-\sigma^2\int_{-\infty}^{\infty}(s-\mu)f(s)ds\}$$

$$=0$$

The first inequality follows from the concavity of $u(\cdot)$ and the normalization imposed; the last equality follows from the fact that R is normally distributed and, hence, $E[(R-E[R])^k]=0$ for k odd.

3.

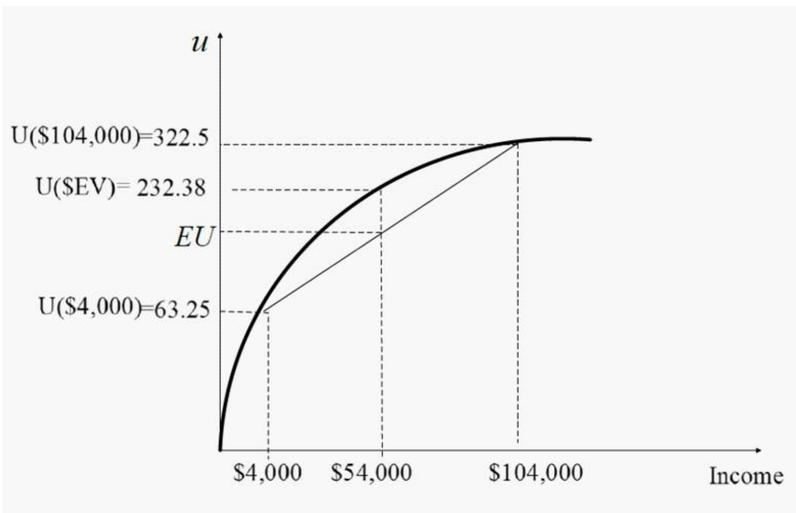
(a) The expected value of accepting the second wage offer is:

$$EV_{Second} = 0.5(\$4{,}000)+0.5(\$104{,}000)=2{,}000+52{,}000 = 54{,}000$$

(b) The expected utility is

$$EU_{Second}=0.5\sqrt{4{,}000}+0.5\sqrt{104{,}000}=192.87$$

(c) The figure below depicts the decision maker's concave utility function, the utility of the first (certain) wage offer of \$54,000 (232.38), the utility of the second (risky) wage offer by separately identifying the utility when the salary is only \$4,000 (63.25) and that when the worker receives the bonus, \$104,000 (322.5). The figure also depicts the expected utility from accepting the second wage offer, which is graphically illustrated by the midpoint of the line connecting the utility in the case in which the decision maker only receives \$4,000 and when he receives \$104,000.



$$\text{Utility function } u(x) = \sqrt{x}$$

(d) We know that the general expression of the risk premium (RP) of a lottery is

$$pu(x_1) + (1-p)u(x_2) = u(EV - RP)$$

Since the left hand side is just the expected utility from the lottery, EU, this expression can be move compactly written as

$$EU = u(EV - RP)$$

Given that we know $EU = 192,87$ from part (b) and that $EV = 54{,}000$ from part (a),

$$192,87 = \sqrt{54{,}000 - RP}$$

Squaring both sides of this equation and rounding to the nearest integer, yields

$$37{,}199 = 54{,}000 - RP \iff RP = \$16{,}801$$

In order to intuitively understand the risk premium of a lottery, consider a decision maker who is offered the expected value of the lottery ($54, 000), with an associated utility of 232.38 with certainty, or the possibility of playing the lottery (where he obtains an expected utility of 192.87). Needless to say, this risk adverse individual would prefer the expected value of the lottery instead which, for convenience, coincides with the first wage offered. The risk premium hence measures by how much we need to reduce the certain wage offer of $54,000 in order to make this individual become indifferent between a risk- less offer (of $54,000 - $16,801 = $37,199), or the expected utility of playing the lottery. In other words, a salary below $37,199, despite being certain, would induce the individual decision maker to prefer the risky second wage offer.

(e) The certain amount of money that would make you exactly indifferent between the utility form this certain payment and the utility from accepting the second (uncertain) wage offer is the so-called *Certainty Equivalent*. As described in our above discussion, the certainty equivalent is obtained from subtracting the risk premium to the certain amount (first wage offer),

$$Certainty\ Equivalent = \$54{,}000 - \$16{,}801 = \$37{,}199$$

4.

Arrow-Pratt coefficient is $r_A(x,u) = -\frac{u''(x)}{u'(x)}$

Hence, $u'(x) = (\alpha + \beta x)^{-\frac{1}{\beta}}$ and $u''(x) = -(\alpha + \beta x)^{-\frac{(1+\beta)}{\beta}}$. Therefore, we have that

$$r_A(x,u) = -\frac{(-(\alpha + \beta x)^{-\frac{(1+\beta)}{\beta}})}{(\alpha + \beta x)^{-\frac{1}{\beta}}}$$

$$r_A(x,u) = \frac{1}{(\alpha + \beta x)}$$

Which is decreasing in wealth as long as $\beta > 0$.