# Homework #2 (Due Wednesday September 12th, 2018)

1. Consider a consumer with utility function $u(x_1, x_2, x_3) = x_1 x_2 x_3$, and income $w$.

   (a) Set up the consumer's utility maximization problem and find the Walrasian demands for each good.

   (b) Let $x_1 + \frac{p_2}{p_1} x_2 = x_c$ denote the units of a composite good. Set up the consumer's utility maximization problem again, but now in terms of the composite good $x_c$. Find the Walrasian demand function for the composite good $x_c$.

   (c) Show that the Walrasian demands you found in parts (a) and (b) are equivalent.

2. Consider that the consumer exhibits Cobb-Douglas utility function $u(x_i, x_j) = x_i^{\alpha} x_j^{\beta}$, where $\alpha, \beta > 0$. For simplicity, you can assume that $\alpha + \beta = 1$.

   (a) Find his Walrasian demand for each good.

   (b) Identify the own-price elasticity $\varepsilon_{ii} \equiv \frac{\partial x_i(p,w)}{\partial p_i} \frac{p_i}{x_i(p,w)}$, the cross-price elasticity $\varepsilon_{ij} \equiv \frac{\partial x_i(p,w)}{\partial p_j} \frac{p_j}{x_i(p,w)}$, the income elasticity $\eta_i \equiv \frac{\partial x_i(p,w)}{\partial w} \frac{w}{x_i(p,w)}$ for the Walrasian demand $x_i(p, w)$ you found in part (a) of the exercise. Interpret your results.

   (c) Find his Hicksian (compensated) demand, $h_i(p, u)$, for every good $i = \{1, 2\}$.

   (d) Identify the own-price elasticity, $\varepsilon_{ii}^C \equiv \frac{\partial h_i(p,w)}{\partial p_i} \frac{p_i}{h_i(p,w)}$, and cross-price elasticity, $\varepsilon_{ij}^C \equiv \frac{\partial h_i(p,w)}{\partial p_j} \frac{p_j}{h_i(p,w)}$, using the Hicksian (compensated) demand, where the superscript $C$ denotes "compensated" demand. Interpret your findings.

3. Fran has a monthly income of \$60. She spends her money making telephone calls (measured in minutes) at a price $p_x$ and on other composite good $y$, whose price has been normalized to one, i.e., $p_y = \$1$. Her mobile phone company offers her two plans: Plan I, in which she pays no monthly fee and makes calls for \$0.50 per minute; or Plan II in which she pays a \$20 monthly fee and benefits from cheaper phone calls at \$0.20 per minute.

   (a) Depict Fran's budget constraint under each of the two plans, with the number of phone calls (good $x$) in the horizontal axis and the composite good (good $y$) in the vertical. Identify the intersection point between these two plans.

   (b) If Fran decides that plan I is better for her, what is the set of baskets she may purchase if her behavior is consistent with WARP.

4. Consider a continuous and strictly increasing utility function $u : \mathbb{R}_+^N \to \mathbb{R}$, and a vector of positive prices $p >> 0$.

   (a) Show that the expenditure function $e(p, u)$ is concave in prices.

   (b) Show that the expenditure function $e(p, u)$ is strictly increasing in the utility level that the consumer seeks to reach, $u$.