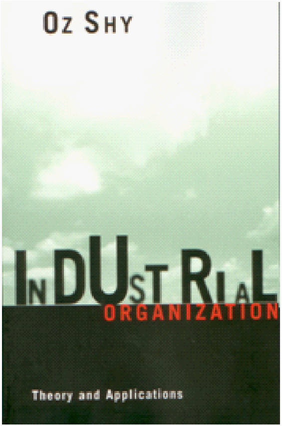
Introduction to Game Theory



OZ SHY

INDUSTRIAL ORGANIZATION

Theory and Applications

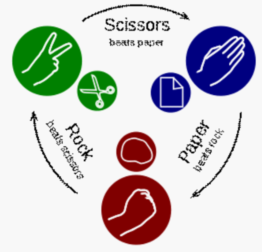
Basic Concepts in Noncooperative Game Theory

- Actions (welfare or profits)
- Help us to analyze industries with few firms
- What are the firms' actions?
- Two types of games:

1. Normal Form Game
2. Extensive Form game

- Two types of actions: Pure and Mixed
- Information: Perfect and Imperfect

Examples of Noncooperative Game Theory



Normal Form Games

- Definition 2.1: A normal form game:

1. N players whose names are listed in the set $I \equiv \{1, 2, ..., N\}$
2. Each player i, $i \in I$, has an action set A^i, where $A^i = \{a^i_1, a^i_2, ..., a^i_{k_i}\}$
3. List of actions chosen by each player: $a \equiv (a^1, a^2, ..., a^i, ..., a^N)$
4. Each player has a payoff function $\pi^i \in \mathbb{R}$

Normal Form Game

- "Peace-War Game" (Prisoners' Dilemma)
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		Country 2	
		War	Peace
Country 1	War	1, 1	3, 0
	Peace	0, 3	2, 2

- Let us apply Definition 2.1.........

Equilibrium Concepts

- We would like to obtain one outcome (unique eq.)
- Outcome of the game: $a \equiv (a^1, a^2, ..., a^i, ..., a^N)$
- $a^{-i} \equiv (a^1, ..., a^{-i}, a^{i+1}, ..., a^N)$
    - let's talk about a^{-i}
- Hence, an outcome a can be expresses as $a \equiv (a^i, a^{-i})$

Equilibrium in dominant actions

- Definition: A particular action $\tilde{a}^i \in A^i$ is said to be a *dominant* action for player i if **no matter what all other players are playing** $\tilde{a}^i$ always maximizes i's payoff

$$\pi^i(\tilde{a}^i, a^{-i})$$

for every $a^i \in A^i$.

- Example: War-Peace game
- What is a Dominant Strategy for player 1?

Country 2

Country 1	War	Peace
War	1, 1	3, 0
Peace	0, 3	2, 2

- Note that an outcome is always composed by a Dominant Strategy

Payoff matrix (Normal Form Game)

		Firm B	
Firm A		Low Prices	High Prices
	Low Prices	5, 5	9, 1
	High Prices	1, 9	7, 7

- *Low prices* yield a higher payoff than *high prices* both when a firm's rival chooses *low prices* and when it selects *high prices*
    - *Low prices* is strictly **dominant strategy** for both firms
    - *High prices* is referred to as a strictly **dominated strategy**

- A strictly dominated strategy can be deleted from the set of strategies a rational player would use.
- This helps to reduce the number of strategies to consider as optimal for each player.
- In the above payoff matrix, both firms will select “low prices” in the unique equilibrium of the game.
- However, games do not always have a strictly dominated strategy.

Battle of the Sexes (coordination games)

Rachel

Jacob	Opera	Football
Opera	2, 1	0, 0
Football	0, 0	1, 2

Battle of the Sexes (coordination games)

- No Dominant Strategies!
- Hence, there does not exist an equilibrium in dominant actions

Nash Equilibrium (NE)

- Definition: An outcome $\widehat{a} = (\widehat{a}^1, \widehat{a}^2, ..., \widehat{a}^i, ..., \widehat{a}^N)$ is said to be a *NE* if no player would find it beneficial to deviate provided that all other players do not deviate from their strategies played at the Nash outcome

$$\pi^i(\widehat{a}^i, \widehat{a}^{-i}) \geq \pi^i(a^i, \widehat{a}^{-i})$$

  for every $a^i \in A^i$.

- An equilibrium in dominant action is a *NE* but a $NE \nRightarrow eq. D.A$

Nonexistence of a Nash Equilibrium

After 30 years of marriage................ ;)

		Rachel	
		Opera	Football
Jacob	Opera	$\underline{2}, 0$	$0, \underline{2}$
	Football	$0, \underline{1}$	$\underline{1}, 0$

Best-Response functions to solve for NE

- Definition: in a two-player game, the BRF of player i is the function $R^i(a^j)$, that for every given action a^j of player j assigns an action $a^i = R^i(a^j)$ that maximizes player i's payoff $\pi^i(a^i, a^j)$
- Example: Battle of the sexes

Battle of the Sexes (coordination games)

Rachel

Jacob

	Opera	Football
Opera	2, 1	0, 0
Football	0, 0	1, 2

- Example Battle of the sexes
- $$R^J(a^R) = \begin{cases} Opera \text{ if } a^R = Opera \\ Football \text{ if } a^R = Football \end{cases}$$
- $$R^R(a^J) = \begin{cases} Opera \text{ if } a^J = Opera \\ Football \text{ if } a^J = Football \end{cases}$$
- Then if $\widehat{a}$ is a NE, then $\widehat{a}^i = R^i(\widehat{a}^{-i})$ for every player i.