Advanced Microeconomic Theory

Chapter 7: Monopoly
Outline

• Barriers to Entry
• Profit Maximization under Monopoly
• Welfare Loss of Monopoly
• Multiplant Monopolist
• Price Discrimination
• Advertising in Monopoly
• Regulation of Natural Monopolies
• Monopsony
Barriers to Entry
Barriers to Entry

• *Entry barriers*: elements that make the entry of potential competitors either impossible or very costly.

• Three main categories:
  1) *Legal*: the incumbent firm in an industry has the legal right to charge monopoly prices during the life of the patent
     – *Example*: newly discovered drugs
Barriers to Entry

2) **Structural**: the incumbent firm has a cost or demand advantage relative to potential entrants.
   – superior technology
   – a loyal group of customers
     ▪ positive network externalities (Facebook, eBay)

3) **Strategic**: the incumbent monopolist has a reputation of fighting off newcomers, ultimately driving them off the market.
   – price wars
Profit Maximization under Monopoly
Profit Maximization

• Consider a demand function \( x(p) \), which is continuous and strictly decreasing in \( p \), i.e., \( x'(p) < 0 \).

• We assume that there is price \( \bar{p} < \infty \) such that \( x(p) = 0 \) for all \( p > \bar{p} \).

• Also, consider a general cost function \( c(q) \), which is increasing and convex in \( q \).
Profit Maximization

- $\bar{p}$ is a “choke price”
- No consumers buy positive amounts of the good for $p > \bar{p}$.

\[ x(p) = 0 \text{ for all } p > \bar{p} \]

\[ x'(p) < 0 \]
Profit Maximization

• Monopolist’s decision problem is

$$\max_p px(p) - c(x(p))$$

• Alternatively, using $x(p) = q$, and taking the inverse demand function $p(q) = x^{-1}(p)$, we can rewrite the monopolist’s problem as

$$\max_{q \geq 0} p(q)q - c(q)$$
Profit Maximization

• Differentiating with respect to $q$,
  
  \[ p(q^m) + p'(q^m)q^m - c'(q^m) \leq 0 \]

• Rearranging,

  \[ p(q^m) + p'(q^m)q^m \leq c'(q^m) \]

  \[ MR = \frac{d[p(q)q]}{dq} \leq \frac{MC}{MC} \]

  with equality if $q^m > 0$.

• Recall that total revenue is $TR(q) = p(q)q$
Profit Maximization

• In addition, we assume that $p(0) \geq c'(0)$.
  – That is, the inverse demand curve originates above the marginal cost curve.
  – Hence, the consumer with the highest willingness to pay for the good is willing to pay more than the variable costs of producing the first unit.

• Then, we must be at an interior solution $q^m > 0$, implying

$$\frac{p(q^m)}{MR} + p'(q^m)q^m = c'(q^m)$$

$$\text{MC}$$
Profit Maximization

• Note that

\[ p(q^m) + \frac{p'(q^m)q^m}{2} = c'(q^m) \]

• Then, \( p(q^m) > c'(q^m) \), i.e., monopoly price \( > MC \)

• Moreover, we know that in competitive equilibrium \( p(q^*) = c'(q^*) \).

• Then, \( p^m > p^* \) and \( q^m < q^* \).
Profit Maximization
Profit Maximization

• Marginal revenue in monopoly

\[ MR = p(q^m) + p'(q^m)q^m \]

MR describes two effects:

– A direct (positive) effect: an additional unit can be sold at \( p(q^m) \), thus increasing revenue by \( p(q^m) \).

– An indirect (negative) effect: selling an additional unit can only be done by reducing the market price of all units (the new and all previous units), ultimately reducing revenue by \( p'(q^m)q^m \).

  ▪ Inframarginal units – initial units before the marginal increase in output.
Profit Maximization

• Is the above FOC also sufficient?
  – Let’s take the FOC \( p(q^m) + p'(q^m)q^m - c'(q^m) \), and differentiate it wrt \( q \),
    \[ p'(q) + p'(q) + p''(q)q - c''(q) \leq 0 \]
    \[ \frac{dMR}{dq} \quad \frac{dMC}{dq} \]
  – That is, \( \frac{dMR}{dq} \leq \frac{dMC}{dq} \).
  – Since MR curve is decreasing and MC curve is weakly increasing, the second-order condition is satisfied for all \( q \).
Profit Maximization

\[ p^m \]

\[ q^m \]

\[ MC(q) \]

\[ x(p) \]

\[ MR(q) \]
Profit Maximization

• What would happen if MC curve was decreasing in $q$ (e.g., concave technology given the presence of increasing returns to scale)?
  – Then, the slopes of MR and MC curves are both decreasing.
  – At the optimum, MR curve must be steeper MC curve.
Profit Maximization

\[ p^m \]

\[ q^m \]
Profit Maximization: Lerner Index

• Can we re-write the FOC in a more intuitive way? Yes.
  
  – Just take $MR = p(q) + p'(q)q = p + \frac{\partial p}{\partial q} q$ and multiply by $\frac{p}{p}$,
  
  $$MR = p \frac{p}{p} + \frac{\partial p}{\partial q} \frac{q}{p} = p + \frac{1}{\varepsilon_d} p$$
  
  – In equilibrium, $MR(q) = MC(q)$. Hence, we can replace MR with MC in the above expression.
Profit Maximization: Lerner Index

• Rearranging yields

\[
\frac{p - MC(q)}{p} = 1 - \frac{1}{\varepsilon_d}
\]

• This is the **Lerner index** of market power
  – The price mark-up over marginal cost that a monopolist can charge is a function of the elasticity of demand.

• Note:
  – If \( \varepsilon_d \to \infty \), then \( \frac{p - MC(q)}{p} \to 0 \implies p = MC(q) \)
  – If \( \varepsilon_d \to 0 \), then \( \frac{p - MC(q)}{p} \to \infty \implies \text{substantial mark-up} \)
Profit Maximization: Lerner Index

• The Lerner index can also be written as

\[ p = \frac{MC(q)}{1 + \frac{1}{\varepsilon_d}} \]

which is referred to as the \textit{Inverse Elasticity Pricing Rule} (IEPR).

• \textbf{Example} (Perloff, 2012):
  
  – Prilosec OTC: \( \varepsilon_d = -1.2 \). Then price should be \( p = \frac{MC(q)}{1 + \frac{1}{-1.2}} = 5.88MC \)

  – Designed jeans: \( \varepsilon_d = -2 \). Then price should be \( p = \frac{MC(q)}{1 + \frac{1}{-2}} = 2MC \)
Profit Maximization: Lerner Index

• *Example 1* (linear demand):
  – Market inverse demand function is
    \[ p(q) = a - bq \]
    where \( b > 0 \)
  – Monopolist’s cost function is \( c(q) = cq \)
  – We usually assume that \( a > c \geq 0 \)
    - To guarantee \( p(0) > c'(0) \)
    - That is, \( p(0) = a - b0 = a \) and \( c'(q) = c \), thus implying \( c'(0) = c \)
Profit Maximization: Lerner Index

• **Example 1** (continued):
  
  – Monopolist’s objective function
  
  \[ \pi(q) = (a - bq)q - cq \]
  
  – FOC: \( a - 2bq - c = 0 \)
  
  – SOC: \( -2b < 0 \) (concave)
    
    ▪ Note that as long as \( b > 0 \), i.e., negatively sloped demand function, profits will be concave in output.
    
    ▪ Otherwise (i.e., Giffen good, with positively sloped demand function) profits will be convex in output.
Profit Maximization: Lerner Index

• **Example 1** (continued):
  
  – Solving for the optimal $q^m$ in the FOC, we find monopoly output
    
    $$q^m = \frac{a - c}{2b}$$

  – Inserting $q^m = \frac{a-c}{2b}$ in the demand function, we obtain monopoly price
    
    $$p^m = a - b \left(\frac{a - c}{2b}\right) = \frac{a + c}{2}$$

  – Hence, monopoly profits are
    
    $$\pi^m = p^m q^m - c q^m = \frac{a - c}{4b}$$
Profit Maximization: Lerner Index

• **Example 1** (continued):
Profit Maximization: Lerner Index

• **Example 1** (continued):

  – Non-constant marginal cost
  – The cost function is convex in output
    \[ c(q) = cq^2 \]
  – Marginal cost is
    \[ c'(q) = 2cq \]

![Graph showing profit maximization](image)
Profit Maximization: Lerner Index

- **Example 2** (Constant elasticity demand):
  - The demand function is \( q(q) = Ap^{-b} \)
  - We can show that \( \varepsilon(q) = -b \) for all \( q \), i.e.,
    \[
    \varepsilon(q) = \frac{\partial q(p)}{\partial p} \frac{p}{q} = \left( -b \right) A p^{-b-1} \frac{p}{\frac{\partial q(p)}{\partial p} \frac{p}{p/q}}
    \]
    \[
    = -b \frac{p^{-b}}{p} \frac{p}{p^{-b}} = -b
    \]
Profit Maximization: Lerner Index

• **Example 2** (continued):
  
  – We can now plug \( \varepsilon(q) = -b \) into the Lerner index,

\[
p^m = \frac{c}{1 - \frac{1}{\varepsilon(q)}} = \frac{c}{1 + \frac{1}{b}}
\]

  – That is, price is a constant mark-up over marginal cost.
Welfare Loss of Monopoly
Welfare Loss of Monopoly

- Welfare comparison for perfect competition and monopoly.

\[ \int_{q^m}^{q^*} [p(s) - c'(s)] \, ds \]
Welfare Loss of Monopoly

• Consumer surplus
  – Perfect competition: $A+B+C$
  – Monopoly: $A$

• Producer surplus:
  – Perfect competition: $D+E$
  – Monopoly: $D+B$

• Deadweight loss of monopoly: $C+E$

$$DWL = \int_{q_m}^{q^*} [p(s) - c'(s)]ds$$

• DWL decreases as demand and/or supply become more elastic.
Welfare Loss of Monopoly

- Infinitely elastic demand
  \[ p'(q) = 0 \]
- The inverse demand curve becomes totally flat.
- Marginal revenue coincides with inverse demand:
  \[ MR(q) = p(q) + 0 \cdot q = p(q) \]
- Profit-maximizing \( q \)
  \[ MR(q) = MC(q) \implies p(q) = MC(q) \]
- Hence, \( q^m = q^* \) and \( DWL = 0 \).
Welfare Loss of Monopoly

• **Example** (Welfare losses and elasticity):
  – Consider a monopolist with constant marginal and average costs, \( c'(q) = c \), who faces a market demand with constant elasticity
    \[
    q(p) = p^e
    \]
    where \( e \) is the price elasticity of demand (\( e < -1 \))
  – Perfect competition: \( p_c = c \)
  – Monopoly: using the IEPR
    \[
    p^m = \frac{c}{1 + \frac{1}{e}}
    \]
Welfare Loss of Monopoly

• **Example** (continued):
  – The consumer surplus associated with any price \( p_0 \) can be computed as
    \[
    CS = \int_{p_0}^{\infty} Q(P)dp = \int_{p_0}^{\infty} p^e dp = \left. \frac{p^{e+1}}{e+1} \right|_{p_0}^{\infty} - \frac{p_0^{e+1}}{e+1}
    \]
  – Under perfect competition, \( p_c = c \),
    \[
    CS = -\frac{c^{e+1}}{e+1}
    \]
  – Under monopoly, \( p^m = \frac{c}{1+1/e} \),
    \[
    CS_m = -\left( \frac{c}{1+1/e} \right)^{e+1} \frac{e+1}{e+1}
    \]
Welfare Loss of Monopoly

• *Example* (continued):
  – Taking the ratio of these two surpluses
    \[
    \frac{CS_m}{CS} = \left( \frac{1}{1 + 1/e} \right)^{e+1}
    \]
  – If \( e = -2 \), this ratio is \( \frac{1}{2} \)
    ▪ CS under monopoly is half of that under perfectly competitive markets
Welfare Loss of Monopoly

• **Example** (continued):
  
  – The ratio \( \frac{CS_m}{CS} = \left( \frac{1}{1 + 1/e} \right)^{e+1} \) decreases as demand becomes more elastic.
Welfare Loss of Monopoly

• **Example** (continued):
  – Monopoly profits are given by
    \[
    \pi^m = p^m q^m - cq^m = \left( \frac{c}{1 + 1/e} - c \right) q^m
    \]
    where \( q^m(p) = p^e = \left( \frac{c}{1 + 1/e} \right)^e \).
  – Re-arranging,
    \[
    \pi^m = \left( \frac{-c/e}{1 + 1/e} \right) \left( \frac{c}{1 + 1/e} \right)^e
    \]
    \[
    = - \left( \frac{c}{1 + 1/e} \right)^{e+1} \cdot \frac{1}{e}
    \]
Welfare Loss of Monopoly

• **Example** (continued):
  
  – To find the transfer from CS into monopoly profits that consumers experience when moving from a perfectly competitive market to a monopoly, divide monopoly profits by the competitive CS profits:
  
  \[
  \frac{\pi^m}{CS} = \left( \frac{e+1}{e} \right) \left( \frac{1}{1+1/e} \right)^{e+1} = \left( \frac{e}{1+e} \right)^e
  \]
  
  – If \( e = -2 \), this ratio is \( \frac{1}{4} \)
    
    ▪ One fourth of the consumer surplus under perfectly competitive markets is transferred to monopoly profits
Welfare Loss of Monopoly

• More social costs of monopoly:
  – Excessive R&D expenditure (patent race)
  – Persuasive (not informative) advertising
  – Lobbying costs (different from bribes)
  – Resources to avoid entry of potential firms in the industry
Comparative Statics
Comparative Statics

• We want to understand how $q^m$ varies as a function of monopolist’s marginal cost
Comparative Statics

• Formally, we know that at the optimum, \( q^m(c) \), the monopolist maximizes its profits
\[
\frac{\partial \pi(q^m(c), c)}{\partial q^m} = 0
\]

• Differentiating wrt \( c \), and using the chain rule,
\[
\frac{\partial^2 \pi(q^m(c), c)}{\partial q^2} \frac{dq^m(c)}{dc} + \frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c} = 0
\]

• Solving for \( \frac{dq^m(c)}{dc} \), we have
\[
\frac{dq^m(c)}{dc} = -\frac{\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c}}{\frac{\partial^2 \pi(q^m(c), c)}{\partial q^2}}
\]
Comparative Statics

• **Example:**
  
  – Assume linear demand curve $p(q) = a - bq$
  
  – Then, the cross-derivative is

  $$
  \frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c} = \frac{\partial}{\partial c} \left( \frac{\partial}{\partial q} \left[ (a - bq)q - cq \right] \right)
  = \frac{\partial}{\partial c} [a - 2bq - c] = -1
  $$

  and

  $$
  \frac{dq^m(c)}{dc} = - \frac{\frac{\partial^2 \pi(q^m(c), c)}{\partial q^2}}{\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c}} = - \frac{-1}{-2b} < 0
  $$
Comparative Statics

• **Example** (continued):
  
  – That is, an increase in marginal cost, \( c \), decreases monopoly output, \( q^m \).
  
  – Similarly for any other demand.
  
  – Even if we don’t know the accurate demand function, but know the sign of

\[
\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c}
\]
Comparative Statics

- **Example** (continued):
  - Marginal costs are increasing in $q$
  - For convex cost curve $c(q) = cq^2$, monopoly output is
    \[ q^m(c) = \frac{a}{2(b + c)} \]
  - Here,
    \[ \frac{dq^m(c)}{dc} = -\frac{a}{2(b + c)^2} < 0 \]
Comparative Statics

- **Example** (continued):
  - Constant marginal cost
  - For the constant-elasticity demand curve $q(p) = p^e$, we have $p^m = \frac{c}{1+1/e}$ and
    $$q^m(c) = \left(\frac{ec}{1+e}\right)^e$$
  - Here,
    $$\frac{dq^m(c)}{dc} = e \cdot \frac{ec}{c(1+e)}$$
    $$= \frac{e^2}{c} q^m < 0$$
Multiplant Monopolist
Multiplant Monopolist

• Monopolist produces output $q_1, q_2, ..., q_N$ across $N$ plants it operates, with total costs $TC_i(q_i)$ at each plant $i = \{1, 2, ..., N\}$.

• Profits-maximization problem

$$\max_{q_1, \ldots, q_N} \left[ a - b \sum_{i=1}^{N} q_i \right] \sum_{i=1}^{N} q_i - \sum_{i=1}^{N} TC_i(q_i)$$

• FOCs wrt production level at every plant $j$

$$a - 2b \sum_{i=1}^{N} q_i - MC_j(q_j) = 0$$

$$\iff MR(Q) = MC_j(q_j)$$

for all $j$.
Multiplant Monopolist

• Multiplant monopolist operating two plants with marginal costs $MC_1$ and $MC_2$. 

![Graph of Multiplant Monopolist](image)
Multiplant Monopolist

- Total marginal cost is $MC_{total} = MC_1 + MC_2$ (i.e., horizontal sum)
- $Q_{total}$ is determined by $MR = MC_{total}$ (i.e., point A)
- Mapping $Q_{total}$ in the demand curve, we obtain price $p^m$ (both plants sell at the same price)
- At the MC level for which $MR = MC_{total}$ (i.e., point A), extend a line to the left crossing $MC_1$ and $MC_2$.
- This will give us output levels $q_1$ and $q_2$ that plants 1 and 2 produce, respectively.
Multiplant Monopolist

• **Example 1** (symmetric plants):
  – Consider a monopolist operating $N$ plants, where all plants have the *same* cost function $TC_i(q_i) = F + cq_i^2$. Hence, all plants produce the same output level $q_1 = q_2 = \cdots = q_N = q$ and $Q = Nq_j$. The linear demand function is given by $p = a - bQ$.
  – FOCs:
    
    \[a - 2b \sum_{j=1}^{N} q_j = 2cq_j \quad \text{or} \quad a - 2bNq_j = 2cq_j\]

    \[q_j = \frac{a}{2(bN + c)}\]
Multiplant Monopolist

• **Example 1** (continued):
  – Total output produced by the monopolist is
    \[ Q = Nq_j = \frac{Na}{2(bN + c)} \]
  and market price is
    \[ p = a - bQ = a - b \frac{Na}{2(bN + c)} = \frac{a(bN + 2c)}{2(bN + c)} \]
  – Hence, the profits of every plant \( j \) are
    \[ \pi_j = \frac{a^2}{4(bN + c)} - F \]
  with total profits of
    \[ \pi_{total} = \frac{Na^2}{4(bN + c)} - NF \]
Multiplant Monopolist

• **Example 1** (continued):

  – The optimal number of plants $N^*$ is determined by

  \[
  \frac{d\pi_{total}}{dN} = \frac{a^2}{4} \frac{c}{(bN + c)^2} - F = 0
  \]

  and solving for $N$

  \[
  N^* = \frac{1}{b} \left( \frac{a}{2} \sqrt{\frac{c}{F}} - c \right)
  \]

  – $N^*$ is decreasing in the fixed costs $F$, and also decreasing in $c$, as long as $a < 4\sqrt{cF}$. 

Advanced Microeconomic Theory
Multiplant Monopolist

• *Example 1* (continued):
  
  – Note that when $N = 1$, $Q = q^m$ and $p = p^m$.
  
  – Note that an increase in $N$ decreases $q_j$ and $\pi_j$. 
Multiplant Monopolist

- **Example 2** (asymmetric plants):
  - Consider a monopolist operating two plants with marginal costs $MC_1(q_1) = 10 + 20q_1$ and $MC_2(q_2) = 60 + 5q_2$, respectively. A linear demand function is given by $p(Q) = 120 - 3Q$.
  - Note that $MC_{total} \neq MC_1(q_1) + MC_2(q_2)$
    - This is a vertical (not a horizontal) sum.
  - Instead, first invert the marginal cost functions
    
    $MC_1(q_1) = 10 + 20q_1 \iff q_1 = \frac{MC_1}{20} - \frac{1}{2}$
    
    $MC_2(q_2) = 60 + 5q_2 \iff q_2 = \frac{MC_2}{5} - 12$
Multiplant Monopolist

• **Example 2** (continued):
  
  Second,
  
  \[ Q_{total} = q_1 + q_2 = \frac{MC_{total}}{20} - \frac{1}{2} + \frac{MC_{total}}{5} - 12 \]

  \[ = \frac{1}{4}MC_{total} - 12.5 \]

  Hence, \( MC_{total} = 50 + 4Q_{total} \)

  Setting \( MR(Q) = MC_{total} \), we obtain \( Q_{total} = 7 \) and \( p = 120 - 3 \cdot 7 = 99 \).

  Since \( MR(Q_{total}) = 120 - 6 \cdot 7 = 78 \), then

  \[ MR(Q_{total}) = MC_1(q_1) \implies 78 = 10 + 20q_1 \implies q_1 = 3.4 \]

  \[ MR(Q_{total}) = MC_2(q_2) \implies 78 = 60 + 5q_2 \implies q_2 = 3.6 \]
Price Discrimination
Price Discrimination

• Can the monopolist capture an even larger surplus?

  – Charge $p > p^m$ to those who buy the product at $p^m$ and are willing to pay more.
  – Charge $c < p < p^m$ to those who do not buy the product at $p^m$, but whose willingness to pay for the good is still higher than the marginal cost of production, $c$.
  – With $p^m$ for all units, the monopolist does not capture the surplus of neither of these segments.
Price Discrimination: First-degree

- **First-degree (perfect) price discrimination:**
  - The monopolist charges to every customer his/her maximum willingness to pay for the object.

- **Personalized price:**
  The first buyer pays $p_1$ for the $q_1$ units, the second buyer pays $p_2$ for $q_2 - q_1$ units, etc.
Price Discrimination: First-degree

– The monopolist continues doing so until the last buyer is willing to pay the marginal cost of production.

– In the limit, the monopolist captures all the area below the demand curve and above the marginal cost (i.e., consumer surplus)
Price Discrimination: First-degree

• Suppose that the monopolist can offer a fixed fee, $r^*$, and an amount of the good, $q^*$, that maximizes profits.

• PMP:

$$\max_{r,q} \ r - cq$$
$$\text{s.t. } u(q) \geq r$$

• Note that the monopolist raises the fee $r$ until $u(q) = r$. Hence we can reduce the set of choice variables

$$\max_q \ u(q) - cq$$

• FOC: $u'(q^*) - c = 0$ or $u'(q^*) = c$.
  – *Intuition*: monopolist increases output until the marginal utility that consumers obtain from additional units coincides with the marginal cost of production.
Price Discrimination: First-degree

• Given the level of production $q^*$, the optimal fee is $r^* = u(q^*)$

• *Intuition*: the monopolist charges a fee $r^*$ that coincides with the utility that the consumer obtains from $q^*$

\[
\text{Profits} = \int_0^{q^*} p(q) dq - c(q^*)
\]
Price Discrimination: First-degree

• **Example:**

  – A monopolist faces inverse demand curve $p(q) = 20 - q$ and constant marginal costs $c = \$2$.

  – No price discrimination:

    \[ MR = MC \quad \Rightarrow \quad 20 - 2q = 2 \quad \Rightarrow \quad q^m = 9 \]

    \[ p^m = \$11, \quad \pi^m = \$81 \]

  – Price discrimination:

    \[ p(Q) = MC \quad \Rightarrow \quad 20 - Q = 2 \quad \Rightarrow \quad Q = 18 \]

    \[ \pi = \$162 \]
Price Discrimination: First-degree

• *Example* (continued):
Price Discrimination: First-degree

• Summary:
  – Total output coincides with that in perfect competition
  – Unlike in perfect competition, the consumer does not capture any surplus
  – The producer captures all the surplus
  – Due to information requirements, we do not see many examples of it in real applications
    ▪ Financial aid in undergraduate education ("tuition discrimination")
Price Discrimination: First-degree

- **Example** (two-block pricing):
  - A monopolist faces a inverse demand curve $p(q) = a - bq$, with constant marginal costs $c < a$.
  - Under two-block pricing, the monopolist sells the first $q_1$ units at a price $p(q_1) = p_1$ and the remaining $q_2 - q_1$ units at a price $p(q_2) = p_2$. 

![Diagram showing two-block pricing](image)
Price Discrimination: First-degree

• **Example** (continued):
  
  – Profits from the first $q_1$ units
  
  $$\pi_1 = p(q_1) \cdot q_1 - cq_1 = (a - bq_1 - c)q_1$$

  while from the remaining $q_2 - q_1$ units

  $$\pi_2 = p(q_2) \cdot (q_2 - q_1) - c \cdot (q_2 - q_1)$$
  
  $$= (a - bq_2 - c)(q_2 - q_1)$$

  – Hence total profits are

  $$\pi = \pi_1 + \pi_2$$
  
  $$= (a - bq_1 - c)q_1 + (a - bq_2 - c)(q_2 - q_1)$$
Price Discrimination: First-degree

• **Example** (continued):

  – FOCs:
    \[
    \frac{\partial \pi}{\partial q_1} = a - 2bq_1 - c - a + bq_2 + c = 0
    \]
    \[
    \frac{\partial \pi}{\partial q_2} = -b(q_2 - q_1) + a - bq_2 - c = 0
    \]
  – Solving for \( q_1 \) and \( q_2 \)
    \[
    q_1 = \frac{a - c}{3b}, \quad q_2 = \frac{2(a - c)}{3b}
    \]
  which entails prices of
    \[
    p(q_1) = a - b \cdot \frac{a - c}{3b} = \frac{2a + c}{3}, \quad p(q_2) = \frac{a + 2c}{3}
    \]
  where \( p(q_1) > p(q_2) \) since \( a > c \).
Price Discrimination: First-degree

• **Example** (continued):

  – The monopolist’s profits from each block are

  \[
  \pi_1 = (p(q_1) - c) \cdot q_1
  \]

  \[
  = \left(\frac{2a + c}{3} - c\right) \cdot \frac{a - c}{3b} = \frac{2}{b} \left(\frac{a - c}{3}\right)^2
  \]

  \[
  \pi_2 = (p(q_2) - c)(q_2 - q_1)
  \]

  \[
  = \left(\frac{a + 2c}{3} - c\right) \cdot \left(\frac{2(a - c)}{3b} - \frac{a - c}{3b}\right) = \frac{1}{b} \left(\frac{a - c}{3}\right)^2
  \]

  – Thus, \( \pi = \pi_1 + \pi_2 = \frac{(a-c)^2}{3b} \), which is larger than those arising under uniform pricing, \( \pi^u = \frac{(a-c)^2}{4b} \).
Price Discrimination: Third-degree

• **Third degree price discrimination:**
  – The monopolist charges different prices to two or more groups of customers (each group must be easily recognized by the seller).
    • *Example:* youth vs. adult at the movies, airline tickets
  – Firm’s PMP:
    \[
    \max_{x_1,x_2} p_1(x_1)x_1 + p_2(x_2)x_2 - cx_1 - cx_2
    \]
  – FOCs:
    \[
    p_1(x_1) + p'_1(x_1)x_1 - c = 0 \implies MR_1 = MC
    \]
    \[
    p_2(x_2) + p'_2(x_2)x_2 - c = 0 \implies MR_2 = MC
    \]
  – FOCs coincides with those of a regular monopolist who serves two completely separated markets practicing uniform pricing.
Price Discrimination: Third-degree

- Example: \( p_1(x_1) = 38 - x_1 \) for adults and \( p_2(x_2) = 14 - \frac{1}{4}x_2 \) for seniors, with \( MC = $10 \) for both markets.

\[
\begin{align*}
MR_1(x_1) &= MC \Rightarrow 38 - x_1 = 10 \Rightarrow x_1 = 14 \quad p_1 = $24 \\
MR_2(x_2) &= MC \Rightarrow 14 - \frac{1}{4}x_2 = 10 \Rightarrow x_2 = 8 \quad p_2 = $12
\end{align*}
\]
Price Discrimination: Third-degree

- Using the Inverse Elasticity Pricing Rule (IERP), we can obtain the prices
  \[ p_1(x_1) = \frac{c}{1 – 1/\varepsilon_1} \quad \text{and} \quad p_2(x_2) = \frac{c}{1 – 1/\varepsilon_2} \]
  where \( c \) is the common marginal cost.
- Then, \( p_1(x_1) > p_2(x_2) \) if and only if
  \[ \frac{c}{1 – 1/\varepsilon_1} > \frac{c}{1 – 1/\varepsilon_2} \quad \Rightarrow \quad 1 – \frac{1}{\varepsilon_2} < 1 – \frac{1}{\varepsilon_1} \]
  \[ \Rightarrow \quad \frac{1}{\varepsilon_2} > \frac{1}{\varepsilon_1} \quad \Rightarrow \quad \varepsilon_2 < \varepsilon_1 \]
- **Intuition**: the monopolist charges lower price in the market with more elastic demand.
Price Discrimination: Third-degree

• **Example** (Pullman-Seattle route):
  – The price-elasticity of demand for business-class seats is -1.15, while that for economy seats is -1.52
  – From the IEPR,
    \[
    p_B = \frac{MC}{1 - 1/1.15} \quad \Rightarrow \quad 0.13p_B = MC
    \]
    \[
    p_E = \frac{MC}{1 - 1/1.52} \quad \Rightarrow \quad 0.34p_E = MC
    \]
  – Hence, \(0.13p_B = 0.34p_E\) or \(p_B = 2.63p_E\)
    – Airline maximizes its profits by charging business-class seats a price 2.63 times higher than that of economy-class seats
Price Discrimination: Second-degree

• Second-degree price discrimination:
  – The monopolist cannot observe the type of each consumer (e.g., his willingness to pay).
  – Hence the monopolist offers a menu of two-part tariffs, \((F_L, q_L)\) and \((F_H, q_H)\), with the property that the consumer with type \(i = \{L, H\}\) has the incentive to self-select the two-part tariff \((F_i, q_i)\) meant for him.
Price Discrimination: Second-degree

• Assume the utility function of type $i$ consumer

\[ U_i(q_i, F_i) = \theta_i u(q_i) - F_i \]

where

- $q_i$ is the quantity of a good consumed
- $F_i$ is the fixed fee paid to the monopolist for $q_i$
- $\theta_i$ measures the valuation consumer assigns to the good, where $\theta_H > \theta_L$, with corresponding probabilities $p$ and $1 - p$.

• The monopolist’s constant marginal cost $c$ satisfies $\theta_i > c$ for all $i = \{L, H\}$. 
Price Discrimination: Second-degree

• The monopolist must guarantee that
  1) both types of customers are willing to participate ("participation constraint")
     ▪ the two-part tariff meant for each type of customer provides him with a weakly positive utility level
  2) customers do not have incentives to choose the two-part tariff meant for the other type of customer ("incentive compatibility")
     ▪ type $i$ customer prefers $(F_i, q_i)$ over $(F_j, q_j)$ where $j \neq i$
Price Discrimination: Second-degree

• The participation constraints (PC) are
  \[ \theta_L u(q_L) - F_L \geq 0 \quad PC_L \]
  \[ \theta_H u(q_H) - F_H \geq 0 \quad PC_H \]

• The incentive compatibility conditions are
  \[ \theta_L u(q_L) - F_L \geq \theta_L u(q_H) - F_H \quad IC_L \]
  \[ \theta_H u(q_H) - F_H \geq \theta_H u(q_L) - F_L \quad IC_H \]
Price Discrimination: Second-degree

- Re-arranging the four inequalities, the monopolist’s profit maximization problem becomes:

\[
\max_{F_L, q_L, F_H, q_H} \quad p[F_H - cq_H] + (1 - p)[F_L - cq_L] \\
\theta_L u(q_L) \geq F_L \\
\theta_H u(q_H) \geq F_H \\
\theta_L [u(q_L) - u(q_H)] + F_H \geq F_L \\
\theta_H [u(q_H) - u(q_L)] + F_L \geq F_H
\]
Price Discrimination: Second-degree

• Both $PC_H$ and $IC_H$ are expressed in terms of the fee $F_H$
  – The monopolist increases $F_H$ until such fee coincides with the lowest of $\theta_H u(q_H)$ and $\theta_H [u(q_H) - u(q_L)] + F_L$ for all $i = \{L, H\}$
  – Otherwise, one (or both) constraints will be violated, leading the high-demand customer to not participate
Price Discrimination: Second-degree
Price Discrimination: Second-degree

• **High-demand customer:**
  
  – Let us show that $IC_H$ is binding
  
  – An indirect way to show that

  \[ F_H = \theta_H [u(q_H) - u(q_L)] + F_L \]

  is to demonstrate that $F_H < \theta_H u(q_H)$

  – Proving this by contradiction, assume that

  \[ F_H = \theta_H u(q_H) \]
Price Discrimination: Second-degree

– Then, $IC_H$ can be written as

$$F_H - \theta_H u(q_L) + F_L \geq F_H$$

$$\Rightarrow F_L \geq \theta_H u(q_L)$$

– Combining this result with the fact that $\theta_H > \theta_L$,

$$F_L \geq \theta_H u(q_L) > \theta_L u(q_L)$$

which implies $F_L > \theta_L u(q_L)$

– However, this violates $PC_L$

  • We then reached a contradiction
  • Thus, $F_H < \theta_H u(q_H)$
  • $IC_H$ is binding but $PC_H$ is not.
Price Discrimination: Second-degree

• **Low-demand customer:**
  - Let us show that $PC_L$ binding
  - Similarly as for the high-demand customer, an indirect way to show that
    \[ F_L = \theta_L u(q_L) \]
    is to demonstrate that $F_L < \theta_L [u(q_L) - u(q_H)] + F_H$
  - Proving this by contradiction, assume that
    \[ F_L = \theta_L [u(q_L) - u(q_H)] + F_H \]
Price Discrimination: Second-degree

Then, \( IC_H \) can be written as

\[
\theta_H [u(q_H) - u(q_L)] + \theta_L [u(q_L) - u(q_H)] + F_H = F_H \\
\Rightarrow \theta_H [u(q_H) - u(q_L)] = \theta_L [u(q_L) - u(q_H)] \\
\Rightarrow \theta_H = \theta_L
\]

which violates the initial assumption \( \theta_H > \theta_L \)

- We reached a contradiction
- Thus, \( F_L < \theta_L [u(q_L) - u(q_H)] + F_H \)
- \( PC_L \) is binding but \( IC_L \) is not
Price Discrimination: Second-degree

• In summary:
  – From $PC_L$ binding we have
    \[ \theta_L u(q_L) = F_L \]
  – From $IC_H$ binding we have
    \[ \theta_H [u(q_H) - u(q_L)] + F_L = F_H \]
  – In addition,
    • $PC_L$ binding implies that $IC_L$ holds, and
    • $IC_H$ binding entails that $PC_H$ is also satisfied,
    • That is, all four constraints hold.
Price Discrimination: Second-degree

- The monopolist’s expected PMP can then be written as unconstrained problem, as follows,

\[
\max_{q_L, q_H \geq 0} p \left[ F_H - cq_H \right] + (1 - p) \left[ F_L - cq_L \right] \\
= p \left\{ \theta_H [u(q_H) - u(q_L)] + \frac{F_H}{F_L} - cq_H \right\} \\
\quad + (1 - p) \left\{ \theta_L u(q_L) - cq_L \right\} \\
= p \left\{ \theta_H [u(q_H) - u(q_L)] + \frac{\theta_L u(q_L) - cq_H}{F_L} \right\} \\
\quad + (1 - p)\{\theta_L u(q_L) - cq_L\} \\
= p[\theta_H u(q_H) - (\theta_H - \theta_L)u(q_L) - cq_H] \\
\quad + (1 - p)[\theta_L u(q_L) - cq_L]
\]
Price Discrimination: Second-degree

- **FOC with respect to** $q_H$:
  \[ p\left[\theta_H u'(q_H) - c\right] = 0 \quad \Rightarrow \quad \theta_H u'(q_H) = c \]
  - which coincides with that under complete information.
  - That is, there is not output distortion for high-demand buyer
  - Informally, we say that there is "**no distortion at the top**".

- **FOC with respect to** $q_L$:
  \[ p\left[-(\theta_H - \theta_L)u'(q_L)\right] + (1 - p)[\theta_L u'(q_L) - c] = 0 \]
  which can be re-written as
  \[ u'(q_L)[\theta_L - p\theta_H] = (1 - p)c \]
Price Discrimination: Second-degree

• Dividing both sides by \((1 – p)\), we obtain
  \[
  u'(q_L) \left[ \frac{\theta_L - \theta_H p}{1-p} \right] = c
  \]

• The above expression can alternatively be written as
  \[
  u'(q_L) \left[ \theta_L - \frac{p}{1-p} (\theta_H - \theta_L) \right] = c
  \]
Price Discrimination: Second-degree

- $u'(q_L) \cdot \theta_L$ depicts the socially optimal output $q_{L}^{SO}$, i.e., that arising under complete information.
- The output offered to high-demand customers is socially efficient due to the absence of output distortion for high-type agents.
- The output offered to low-demand customers entails a distortion, i.e., $q_{L} < q_{L}^{SO}$.
- Per-unit price for high-type and low-type differs, i.e., $F_H \neq F_L$.
  - Monopolist practices price discrimination among the two types of customers.
Price Discrimination: Second-degree

• Since constraint $PC_L$ binds while $PC_H$ does not, then only the high-demand customer retains a positive utility level, i.e., $\theta_H u(q_H) - F_H > 0$.

• The firm’s lack of information provides the high-demand customer with an “information rent.”
  - Intuitively, the information rent emerges from the seller’s attempt to reduce the incentives of the high-type customer to select the contract meant for the low type.
  - The seller also achieves self-selection by setting an attractive output for the low-type buyer, i.e., $q_L$ is lower than under complete information.
Price Discrimination: Second-degree

• **Example:**
  
  – Consider a monopolist selling a textbook to two types of graduate students, low- and high-demand, with utility function

  \[ U_i(q_i, F_i) = \frac{q_i^2}{2} - \theta_i q_i - F_i \]

  where \( i = \{L, H\} \) and \( \theta_H > \theta_L \).
  
  – Hence, the UMP of student type \( i \) is

  \[
  \max_{q_i} \frac{q_i^2}{2} - \theta_i q_i - F_i \quad \text{s.t.} \quad pq_i + F_i \leq w_i
  \]

  where \( w_i > 0 \) denotes the student’s wealth.
Price Discrimination: Second-degree

• *Example* (continued):
  
  — By Walras’ law, the constraint binds
    \[ F_i = w_i - pq_i \]
  
  — Then, the UMP can be expressed as
    \[
    \max_{q_i} \frac{q_i^2}{2} - \theta_i q_i - (w_i - pq_i)
    \]
  
  — FOCs wrt $q_i$ yields the direct demand function:
    \[ q_i - \theta_i - p = 0 \text{ or } q_i = \theta_i - p \]
Price Discrimination: Second-degree

• **Example** (continued):
  
  – Assume that the proportion of high-demand (low-demand) students is $\gamma$ ($1 - \gamma$, respectively).

  – The monopolist’s constant marginal cost is $c > 0$, which satisfies $\theta_i > c$ for all $i = \{L, H\}$.

  – Consider for simplicity that $\theta_L > \frac{\theta_H+c}{2}$.

  – This implies that each type of student would buy the textbook, both when the firm practices uniform pricing and when it sets two-part tariffs
    ▪ Exercise.
Advertising in Monopoly
Advertising in Monopoly

• **Advertising**: non-price strategy to capture surplus

• The monopolist must balance the additional demand that advertising entails and its associated costs ($A$ dollars)

• The monopolist solves

$$\max_A p \cdot q(p, A) - TC(q(p, A)) - A$$

where the demand function $q(p, A)$ depends on price and advertising.
Advertising in Monopoly

• Taking FOCs with respect to $A$,

$$p \cdot \frac{\partial q(p,A)}{\partial A} - \frac{\partial TC}{\partial q} \cdot \frac{\partial q(p,A)}{\partial A} - 1 = 0$$

Rearranging, we obtain

$$(p - MC) \frac{\partial q(p,A)}{\partial A} = 1$$

• Let us define the advertising elasticity of demand

$$\varepsilon_{q,A} = \frac{\% \text{ increase in } q}{\% \text{ increase in } A} = \frac{\partial q(p,A)}{\partial A} \cdot \frac{A}{q}$$

Or, rearranging,

$$\varepsilon_{q,A} \cdot \frac{q}{A} = \frac{\partial q(p,A)}{\partial A}$$
Advertising in Monopoly

• We can then rewrite the above FOC as

\[(p - MC) \varepsilon_{q,A} \cdot \frac{q}{A} = 1\]

\[\frac{\partial q(p, A)}{\partial A}\]

• Dividing both sides by \(\varepsilon_{q,A}\) and rearranging

\[p - MC = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{q}\]

• Dividing both sides by \(p\)

\[\frac{p - MC}{p} = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{p \cdot q}\]
Advertising in Monopoly

• From the Lerner index, we know that \( \frac{p - MC}{p} = -\frac{1}{\varepsilon_{q,p}} \). Hence,

\[
-\frac{1}{\varepsilon_{q,p}} = \frac{1}{\varepsilon_{q,A}} \cdot \frac{A}{p \cdot q}
\]

• And rearranging

\[
-\frac{\varepsilon_{q,A}}{\varepsilon_{q,p}} = \frac{A}{p \cdot q}
\]

– The right-hand side represents the advertising-to-sales ratio.

– For two markets with the same \( \varepsilon_{q,p} \), the advertising-to-sales ratio must be larger in the market where demand is more sensible to advertising (higher \( \varepsilon_{q,A} \)).
Advertising in Monopoly

• *Example*:

– If the price-elasticity in a given monopoly market is $\varepsilon_{q,p} = -1.5$ and the advertising-elasticity is $\varepsilon_{q,A} = 0.1$, the advertising-to-sales ratio should be

$$\frac{A}{p \cdot q} = -\frac{0.1}{-1.5} = 0.067$$

– Advertising should account for 6.7% of this monopolist’s revenue.
Regulation of Natural Monopolies
Regulation of Natural Monopolies

• *Natural monopolies*: Monopolies that exhibit *decreasing* cost structures, with the MC curve lying below the AC curve.

• Hence, having a single firm serving the entire market is cheaper than having multiple firms, as aggregate average costs for the entire industry would be lower.
Regulation of Natural Monopolies

• Unregulated natural monopolist maximizes profits at the point where MR=MC, producing $Q_1$ units and selling them at a price $p_1$.

• Regulated natural monopolist will charge $p_2$ (where demand crosses MC) and produce $Q_2$ units.

• The production level $Q_2$ implies a loss of $p_2 - c_2$ per unit.
Regulation of Natural Monopolies

• Dilemma with natural monopolies:
  – abandon the policy of setting prices equal to marginal cost, OR
  – continue applying marginal cost pricing but subsidize the monopolist for his losses

• Solution to the dilemma:
  – A multi-price system that allows for price discrimination
  – Charging some users a high price while maintaining a low price to other users
Regulation of Natural Monopolies

• Multi-price system:
  – a high price $p_1$
  – a low price $p_2$
• Benefit: $(p_1 - c_1)$ per unit in the interval from 0 to $q_1$
• Loss: $(c_2 - p_2)$ per unit in the interval $(q_2 - q_1)$
• The monopolist price discriminates iff
  \[
  (p_1 - c_1)q_1 > (c_2 - p_2)(q_2 - q_1)
  \]
Regulation of Natural Monopolies

• An alternative regulation:
  – allow the monopolist to charge a price above marginal cost that is sufficient to earn a “fair” rate of return on capital investments

• Two difficulties:
  – what is a “fair” rate of return
  – overcapitalization
Regulation of Natural Monopolies

• *Overcapitalization of natural monopolies*: 

  – Suppose a production function of the form \( q = f(k, l) \). An unregulated monopoly with profit function \( pf(k, l) - wl - rk \) has a rate of return on capital, \( r \). Suppose furthermore that the rate of return on capital investments, \( r \), is constrained by a regulatory agency to be equal to \( r_0 \).
Regulation of Natural Monopolies

• PMP:

\[ L = pf(k, l) - wl - rk + \lambda [wl + r_0 k - pf(k, l)] \]

where \( 0 < \lambda < 1 \).

• FOCs:

\[ \frac{\partial L}{\partial l} = pf_l - w + \lambda (w - pf_l) = 0 \]
\[ \frac{\partial L}{\partial k} = pf_k - r + \lambda (r_0 - pf_k) = 0 \]
\[ \frac{\partial L}{\partial \lambda} = wl + r_0 k - pf(k, l) = 0 \]
Regulation of Natural Monopolies

• From the first FOC:
  \[ pf_l = w \]

• From the second FOC:
  \[ (1 - \lambda)pf_k = r - \lambda r_0 \]
  and re-arranging
  \[ pf_k = \frac{r - \lambda r_0}{1 - \lambda} = r - \frac{\lambda(r_0 - r)}{1 - \lambda} \]

  – Since \( r_0 > r \) and \( 0 < \lambda < 1 \), then \( pf_k < r \).
  – Hence, the firm would hire more capital than under unregulated condition, where \( pf_k = r \).
Regulation of Natural Monopolies

• \( pf_k \) is the value of the marginal product of capital
  - It is decreasing in \( k \) (due to diminishing marginal return, i.e., \( f_{kk} < 0 \))

• \( r \) and \( r - \frac{\lambda(r_0 - r)}{1-\lambda} \) are the marginal cost of additional units of capital in the unregulated and regulated, respectively, monopoly

\[ r > r - \frac{\lambda(r_0 - r)}{1-\lambda} \]

• Example: electricity and water suppliers
Regulation of Natural Monopolies

- An alternative illustration of the overcapitalization (Averch-Johnson effect)
- Before regulation, the firm selects \((L^{BR}, K^{BR})\)
- After regulation, the firm selects \((L^{AR}, K^{AR})\), where \(K^{AR} > K^{BR}\) but \(L^{AR} < L^{BR}\)
- The overcapitalization result only captures the substitution effect of a cheaper input.
  - Output effect?
Monopsony
Monopsony

• *Monopsony*: A single buyer of goods and services exercises “buying power” by paying prices below those that would prevail in a perfectly competitive context.

• Monopsony (single buyer) is analogous to that of a monopoly (single seller).

• *Examples*: a coal mine, Walmart Superstore in a small town, etc.
Monopsony

- Consider that the monopsony faces competition in the product market, where prices are given at $p > 0$, but is a monopsony in the input market (e.g., labor services).
- Assume an increasing and concave production function, i.e., $f'(x) > 0$ and $f''(x) \leq 0$.
  - This yields a total revenue of $pf(x)$.
- Consider a cost function $w(x) \cdot x$, where $w(x)$ denotes the inverse supply function of labor $x$.
  - Assume that $w'(x) > 0$ for all $x$.
  - This indicates that, as the firm hires more workers, labor becomes scarce, thus increasing the wages of additional workers.
Monopsony

- The monopsony PMP is
  \[ \max_x pf(x) - w(x)x \]

- FOC wrt the amount of labor services \((x)\) yields
  \[ pf'(x^*) - w(x^*) - w'(x^*)x^* = 0 \]
  \[ \Rightarrow pf'(x^*) = \frac{w(x^*)}{A} + \frac{w'(x^*)x^*}{B} \]
  
  - \(A\): “marginal revenue product” of labor.
  - \(B\): “marginal expenditure” (ME) on labor.
  
  - The additional worker entails a monetary outlay of \(w(x^*)\).
  
  - Hiring more workers make labor become more scarce, ultimately forcing the monopsony to raise the prevailing wage on all inframarginal workers, as captured by \(w'(x^*)x^*\).
Monopsony

- Monopsonist hiring and salary decisions.
  - The marginal revenue product of labor, \( pf'(x) \), is decreasing in \( x \) given that \( f''(x) \leq 0 \).
  - The labor supply, \( w(x) \), is increasing in \( x \) since \( w'(x) > 0 \).
  - The marginal expenditure (ME) on labor lies above the supply function \( w(x) \) since \( w'(x) > 0 \).
  - The monopsony hires \( x^* \) workers at a salary of \( w(x^*) \).
Monopsony

• A deadweight loss from monopsony is

\[ DWL = \int_{x^*}^{x^{PC}} [pf'(x) - w(x)]dx \]

• That is, the area below the marginal revenue product and above the supply curve, between \( x^* \) and \( x^{PC} \) workers.
Monopsony

• We can write the monopsony profit-maximizing condition, i.e., $pf'(x^*) = w(x^*) + w'(x^*)x^*$, in terms of labor supply elasticity, using the following steps:

\[
pf'(x^*) = w(x^*) + \frac{\partial w(x^*)}{\partial x^*} x^* \\
= w(x^*) \left(1 + \frac{\partial w(x^*)}{\partial x^*} \frac{x^*}{w(x^*)}\right)
\]

• And rearranging,

\[
pf'(x^*) = w(x^*) \left(1 + \frac{1}{\frac{\partial x^* w(x^*)}{\partial w x^*}}\right)
\]
Monopsony

• Since \( \frac{\partial x^*}{\partial w} \frac{w(x^*)}{x^*} \) represents the elasticity of labor supply \( \varepsilon \), then

\[
p f'(x^*) = w(x^*) \left( 1 + \frac{1}{\varepsilon} \right)
\]

• Intuitively, as \( \varepsilon \to \infty \), the behavior of the monopsonist approaches that of a pure competitor.
Monopsony

• The equilibrium condition above is also sufficient as long as

\[ pf''(x^*) - 2w'(x^*) - w''(x^*)x^* < 0 \]

• Since \( f''(x^*) < 0, w'(x^*) > 0 \) (by assumption), we only need that either:
  
  a) the supply function is convex, i.e., \( w''(x^*) > 0 \); or
  
  b) if it is concave, i.e., \( w''(x^*) < 0 \), its concavity is not very strong, that is

\[ pf''(x^*) - 2w'(x^*) < w''(x^*)x^* \]
Monopsony

• **Example:**
  
  – Consider a monopsonist with production function $f(x) = ax$, where $a > 0$, and facing a given market price $p > 0$ per unit of output.
  
  – Labor supply is $w(x) = bx$, where $b > 0$.
  
  – The marginal revenue product of hiring an additional worker is
    $$pf'(x) = pa$$
  
  – The marginal expenditure on labor is
    $$w(x) + w'(x)x = bx + bx = 2bx$$
Monopsony

• **Example** (continued):
  
  – Setting them equal to each other, \( pa = 2bx^* \), yields a profit-maximizing amount of labor:
    \[
    x^* = \frac{ap}{2b}
    \]
  
  – \( x^* \) increases in the price of output, \( p \), and in the marginal productivity of labor, \( a \); but decreases in the slope of labor supply, \( b \).
  
  – Sufficiency holds since
    \[
    pf''(x^*) - 2w'(x^*) = p0 - 2a < 0 = w''(x^*)x^*
    \]