# Does Ambiguity Matter for Ex Ante Regulation and Ex Post Liability?[1]

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## Abstract

This paper studies regulation of firms that engage in hazardous activities under ambiguity. A theoretical model is developed where manager's attitudes toward uncertainty are risk averse, risk loving, and ambiguity averse. Ambiguity aversion is modeled using the smooth model of decision making. We show that the uncertainty averse attitudes of managers lead them to over-invest in precaution that reduces hazard. Our main result is in stark contrast to previous findings in which uncertainty causes firms to invest less in precaution. Under this finding, we examine the effectiveness of ex post liability and ex ante regulation with respect to inducing investment in precaution that reduces hazard.

**Keywords:** Ambiguity; ex ante regulation; ex post liability.

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## 1. Introduction

The literature on environmental policy under uncertainty is very extensive. Several papers have analyzed the setting of emission fees, subsidies, or permits when the regulator is unable to observe, for instance, the firm's costs or environmental damage. (see Weitzman (1974), Roberts and Spence (1976), Segerson (1988), Xepapadeas (1991)). However, the study of ambiguity as a type of uncertainty has been overlooked and, as shown in Chambers and Melkonyan (2017), agent behavior and thus policy recommendations can be sensitive to these underlying assumptions.[4] We focus on examining the different attitude toward uncertainty managers may have, how it affects their decision to invest in precaution, and how these attitudes affect environmental policy.

There are different ways to conceive what is meant by uncertainty and how agents deal with that uncertainty. For instance, an agent can face uncertainty that is quantified or unquantified. An agent faces quantified uncertainty when he knows all possible future states and the probability of each possible state. When an agent faces unquantified uncertainty, however, he may not be able to identify all possible states or the probability of each possible state. Knight (1921) was the first to distinguish between these two ideas. When uncertainty is quantified it is not necessary for agents to apply subjective beliefs. They can objectively calculate expected values and variance. If agents facing unquantified uncertainty form subjective beliefs that adhere to the axioms laid out by Savage (1954), then subjective probabilities are equivalent to objective probabilities and agent behavior does not differ between the two cases.[5] When agents do not adhere to Savage's axioms per Ellsberg (1961), they demonstrate ambiguity aversion. The

[4] Chesson and Viscusi (2003) empirically show that ambiguity aversion is important among managers facing decisions under uncertainty.

[5] Ellsberg (1961) criticizes the behavior of individuals implied by Savage's axioms by providing a counterexample when behavior of decision makers was not consistent. Ellsberg demonstrated that agents could violate one of Savage's axioms known as the sure-thing principle.

case where managers are ambiguity averse is examined in this paper in addition to the case where they are not risk neutral.

There is some evidence that decision makers within firms can make decisions that reflect other attitudes toward uncertainty. Kunreuther et al. (1995) finds evidence from a survey of insurance underwriters where they make decisions about insurance premiums that do not reflect risk neutrality or a strictly risk averse attitude. A risk averse agent should charge the same premium whether the probability of the event he is insuring for is ambiguous or not, ceteris paribus. The underwriters, however, would charge a higher premium when faced with ambiguity. Therefore, decisions made by managers can reflect attitudes that are not necessarily risk neutral.[6] Kunreuther (1989) gives a good overview of work that explains why firm and manager incentives may not align.

Klibanoff et al. (2005) develops a smooth model of decision making under ambiguity. They assume that agents form multiple probability distributions over possible states of nature in addition to a distribution over the set of distributions. They evaluate this ambiguity with an increasing concave function which characterizes their attitude toward ambiguity much the same way a utility function characterizes an agent's attitude toward risk. Their results show that ambiguity aversion is distinct from risk aversion because increasing risk aversion does not always affect investment in assets with uncertain returns in the same way as increasing ambiguity aversion. Their method for modeling ambiguity is adopted in this paper.

For illustration, consider a manager tasked with overseeing the operation of an oil rig. Among the many decisions such a manager must make is the amount of time and resources to invest in precaution. Investment reduces both the likelihood of an accident occurring and/or the magnitude of environmental damages in the event an accident occurs. In the absence of a regulatory standard, a legal system that

[6] Fox and Weber (2002) argue that decision makers usually judge the likelihood of an event based on computation intuition with some degree of imprecision and vagueness.

holds firms liable for damages can induce the manager to internalize damages. That is, if the manager expects the firm will be held liable for environmental damages, he will invest in precaution to reduce expected liability. However, if society only relies upon liability to induce investment in precaution, the outcome may be inefficient. Managers' expectation of liability may be low or biased if a lawsuit is unlikely, they anticipate their firm can settle out of court at a relatively low cost, or the courts may be soft on their industry. It is also possible managers may be overly cautious and choose to overinvest. Kolstad et al. (1990) find that an increase in uncertainty causes the firm to invest less in precaution and therefore ex ante regulation is more efficient than liability given sufficiently high uncertainty about the latter. We find that if a manager is uncertainty averse, increased uncertainty about liability causes him to invest even more in precaution, in some settings at a socially excessive level. This implies that increasing uncertainty does not necessarily render one policy option more efficient than the other. It depends on the uncertainty attitudes of managers, the level of uncertainty, and the expected liability. Additionally, it may be difficult for the legal system to set liability that induces optimal investment due to informational asymmetries. Unobservable differences in firms' costs, wealth, or manager's attitudes toward uncertainty can render ex post liability inefficient. Shavell (1984) finds that ex ante regulation should be chosen over liability when heterogeneity of injurer's damages is sufficiently high and when the scenario where the injurer escapes full liability is sufficiently likely. Schmitz (2000) supposes the legal system can impose damages on injurers that exceed the environmental damages, that is damages that are punitive. He finds that regulation is preferable to liability and it is optimal to use both policies when injurer's wealth is heterogeneous and a sufficiently large number of injurers have relatively low wealth. Our results support the idea that sufficient heterogeneity among managers can render joint use more efficient than either policy instituted alone.

If a manager's attitude toward uncertainty leads them to under-invest in precaution for a given level of liability, that precaution level could be increased by rising expected liability or imposing ex ante regulation. Whether ex ante regulation is more efficient than liability can depend on the magnitude of extra costs that enforcement of the regulation introduces or whether estimation of environmental damages is biased. If the cost of enforcement is sufficiently high or estimation of environmental damages by regulators is biased downward, then the optimal level of precaution enforced under ex ante regulation may be lower than the level managers choose under ex post liability. Additionally, heterogeneity of firms' cost of employing precaution can also make a single regulatory standard inefficient.

The results show that risk and ambiguity averse managers over-invest and risk loving managers under-invest in precaution relative to risk neutral managers. That is, in the presence of uncertainty, the more averse managers are, the more they invest in safety. The more uncertainty loving they are the less they invest. Under these conditions, ex post liability may not be a good substitute for ex ante regulation. Even with full information, where risk is calculated and factored into policy, the presence of ambiguity averse managers leads to inefficient investment. Since, the policymaker cannot accurately measure this ambiguity, there is no way to avoid this inefficiency. For this reason, it is important to make the distinction between risk and ambiguity even though aversion to both causes managers to invest more. Furthermore, if the uncertainty attitude of managers is unobservable, regulators and judges can make incorrect assumptions, which could generate additional inefficiencies from regulation and liability. When attempting to influence investment through policy the magnitude of the response by managers depends on their aversion to both types of uncertainty. From a policy perspective, the results indicate that when considering ambiguity there are two possible alternatives to address over-investment in precaution: (1) reduction of penalties, and (2) design of clear laws. While the reduction of penalties can be in some cases politically unfeasible, the policymaker can still rely on (2). The second alternative considers that the reduction of ambiguity can be achieved by making the penalties set by environmental laws clear and easy to interpret. Hence, to reach the socially optimal level of precaution, a regulator can use a combination of the above options when ex-ante and ex-post policies are in place and ambiguity is present.

The rest of the paper goes as follows. Section 2 sets up the model of regulation and shows the socially optimal first best solution. Costly ex ante regulation and estimation error of expected damages are considered and it is demonstrated that the first best solution is unattained. Section 3 analyzes the decisions of risk averse managers and ambiguity averse managers and how ex post liability influences their decisions. We show under which conditions managers overinvest in precaution and when they underinvest. Section 4 provides some discussions about the implications of our results and section 5 concludes.

## 2. Ex Ante Regulation

Consider a production process that has the potential to generate a negative externality.[7] The firm's manager can minimize those damages by investing in precaution level $x$ at a cost $c(x)$, which is increasing in $x$ and convex, i.e., $c'(x) > 0$ and $c''(x) > 0$. Damage $e(x, \upsilon)$ depends on the precaution level $x$ and the random variable $\nu$ which has probability distribution $F_{\upsilon}$ representing the uncertainty surrounding how actions of firms affect the environment.[8] Damages are decreasing and convex in $x$ and increasing in the random variable $\upsilon$.

*Costless enforcement*. First, let us assume that regulation is enforced without cost and expected damages are estimated without error. In this context, the regulator seeks to minimize the cost of ex-ante regulation on the firm and the expected environmental damage. Hence, the regulator solves

$$\min_{x \geq 0} C(x, \upsilon) = c(x) + E_{\upsilon}[e(x, \upsilon)] \qquad (1)$$

where the operator $E_{\upsilon}$ represents the expectation with respect to the random variable $\upsilon$. The optimal precaution level $x^*$ solves (1).

[7] The model embodies cases in which the production process has unwanted consequences in the form of an accident or ongoing uncertain damages.
[8] As $x \to \infty, e(\cdot) \to 0$ and as $x \to 0, e'(\cdot) \to \infty$. This ensures interior solutions for all future calculations.

*Costly enforcement*. Let us now assume that enforcement of regulation is costly, and represented by $r(x)$,[9] but expected damages are accurately estimated. The regulator solves

$$\min_{x \geq 0} C_r(x, v) = c(x) + r(x) + E_v[e(x, v)], \qquad (2)$$

where optimal safety level $x_r$ solves (2), which is characterized by

$$c'(x) + r'(x) = -E_v[e'(x, v)].$$

The optimal precaution level is lower due to the additional marginal cost associated with enforcement, while the expected marginal benefit of precaution remains the same, i.e., $x_r < x^*$. The problem is further compounded if the regulator estimates damages or firms' costs with error. If damages are overestimated the standard will be too high and if underestimated the standard will be too low. Similarly, If the regulator underestimates costs he will set the standard too high and if the regulator overestimates them he will set it too low. Additionally, if we relax the assumption that all firms' cost of precaution is the same and that a regulator only observes the average of firms' cost of precaution, then a singular precaution standard is inefficient. Firms with above average cost of precaution will over-invest and firms with below average cost will under-invest relative to the optimal level of precaution for each firm when a standard based on the average firm is enforced.

For example, suppose there are two firms with cost $c_1(x) = x^2$, and $c_2(x) = x^3$ for firm 1 and 2 respectively, the enforcement cost is $r(x) = x$, and damages are represented by $e(x_i, v) = \frac{v}{x_i}$. Damage $v$ is 1 with 50% probability and 50 with 50% probability. The cost minimization problems associated with precaution for the two firms are

$$\tilde{x}_1 = \operatorname*{argmin}_{x_1} {x_1}^2 + x_1 + \frac{25.5}{x_1} \approx 2.18,$$

[9] For example, enforcement of regulation may require verification that firms are adhering to the safety standard.

$$\tilde{x}_2 = \underset{x_2}{\operatorname{argmin}}\, {x_2}^3 + x_2 + \frac{25.5}{x_2} \approx 1.66.$$

If a regulator, however, imposes a singular precaution standard based on the average cost of precaution, (i.e., $c(x) = x^{2.5}$), he sets the standard according to

$$\tilde{x}_r = \underset{x_r}{\operatorname{argmin}}\, {x_r}^{2.5} + x_r + \frac{25.5}{x_r} \approx 1.86.$$

Hence, precaution level set at 1.86 is too low for firm 1 and too high for firm 2.

In summary, if we assume a regulator can estimate the cost of precaution accurately, there are three factors that contribute to the inefficiency of ex ante regulation. (1) Enforcement cost, (2) error in the calculations of the expected environmental damages, and (3) heterogeneity of costs.

## 3. Ex Post Liability

After an accident occurs or environmental damages caused by the production process becomes known, a judge must determine whether the firm is liable. Managers usually face uncertainty about the judge's verdict. Consequently, they form a subjective expectation about the liability they will face which factors into their decision to invest in precaution. Like Greenwald and Stiglitz (1990), we consider that managers are adversely affected by their firm going into bankruptcy. Their reputation is damaged and they face potentially large job search costs, causing them to not necessarily have risk neutral preferences. In our model, hence, managers are concerned about environmental damage as it affects their firm's liability. Specifically, liability depends on environmental damages and random variable $\omega$, which has probability distribution $F_\omega$.[10] That is, the function $L(e(x, v), \omega)$ determines the liability that the firm faces. Liability is increasing and convex in environmental damages and increasing in the random variable $\omega$. It is

[10] Consider that $\omega$ represents uncertainty surrounding liability, for instance, a manager does not know how a judge will view the selected level of precaution and whether a judge will find the firm negligent. In other words, the manager does not know if the judge will use an objective or a subjective standard. For more details about this interpretation see Calabresi and Klevorick (1985).

assumed that liability is increasing in damages, since it logically follows that a firm's liability burden is higher the larger environmental damages are. In addition, the more punitive a judge's decision is, the larger the firm's liability. Manager $i$'s maximizes his expected utility, as follows

$$\max_{x\geq 0} U_i(x_i, v, \omega) = E_{v,\omega}[u(c(x_i) + L(e(x_i, v), \omega))]$$

where the operator $E_{v,\omega}$ represents the expectation with respect to the random variables $v$ and $\omega$. We assume the random variables $v$ and $\omega$ are not correlated. However, correlation of these variable does not qualitatively affect our results.[11] Hence, the manager's optimal precaution level is

$$\hat{x} = \operatorname*{argmax}_{x_i} U_i(x_i, v, \omega)$$

Comparisons between $\hat{x}$ and the precaution levels $x^*$ and $x_r$ under ex ante regulation depends on the manager's attitude toward uncertainty. To provide meaningful comparisons, we next examine risk neutral, risk averse, and ambiguity averse managers.

### 3.1 Risk Neutral Manager

Let us consider the case in which a judge faces a risk neutral manager.

**Proposition 1.** *If the manager is risk neutral and a judge equates liability with expected environmental damages, $L(e(x_i, v), \omega) = e(x_i, v)$, then the manager chooses the efficient level of precaution.*

Equating liability and damages under risk neutrality makes the manager's optimization problem the same as the social planner's problem. Hence, if we assume that after an accident a judge can accurately assess environmental damages, the establishment of a legal precedent that induces managers to invest in the socially optimal level of precaution only requires equating liability with environmental damages.[12] It is

[11] Since our results do not depend on the second order derivatives of $L(\cdot)$ with respect to the random variables and $\frac{\partial L}{\partial v} > 0$ and $\frac{\partial L}{\partial \omega} > 0$, then correlation between $v$ and $\omega$ only affects the magnitude of liability.
[12] All proofs are relegated to the appendix section.

important to mention that several papers such as Shavell (1984), Kolstad et al. (1990), and Schmitz (2000) have examined cases in which the judge is unable to establish such a clear liability precedent. For instance, Shavell (1984) assumes there exists a positive probability of not being found liable and the judge is unable to impose punitive damages, that is, liability that exceeds environmental damages. In this case, expected liability is always less than expected damages leading to underinvestment in precaution. Kolstad et al. (1990) introduce a mean preserving spread of the probability of being found liable thus leading to expected liability to be less than expected damages. Finally, Schmitz (2000) maintains the same framework but allows judges to impose punitive damages, which in turn allows them to equate expected liability with expected damages. [13] We summarize these findings in corollary 1.

**Corollary 1.** *If a manager is risk neutral and expected liability is lower (higher) than environmental damages, then injurers will under-invest (over-invest) in safety.*

The assumption that a judge can accurately assess environmental damages and establish a clear legal precedent is quite restrictive. We next argue that even under this restrictive setting issues will arise when managers are risk averse or, most important for our study, ambiguity averse. Let us now analyze a numerical example that will help us to clarify the concept of risk neutrality. This example is developed throughout the paper.

**Example 1 (Risk Neutral Manager).** Suppose that the cost function is $c(x_i) = {x_i}^2$, the damage function is represented by $e(x_i, v) = \frac{v}{x_i}$, and that manager $i$'s expected utility is represented by

[13] Judges may not be able to equate liability with expected damages if injurers are heterogeneous and individual characteristics are unobservable, which prevents judges from imposing optimal liability. Examination of these factors can be found in Hiriart et al. (2004) and Rouillon (2008).

$$u(c(x_N) + L(e(x_N, \upsilon), \omega)) = 1000 - E_\upsilon\left[\left(c(x_N) + e(x_N, \upsilon)\right)^{\rho+1}\right].$$[14]

Also, consider that there is a 50 percent chance damages are low, i.e. $\nu_l = 1$, and a 50 percent chance damages are high, i.e. $\nu_h = 50$. When $\rho = 0$ the manager is risk neutral and the solution to the manager's problem is

$$\hat{x}_N = \underset{x_N}{\operatorname{argmax}}\, 1000 - 0.5({x_N}^2 + \tfrac{1}{x_N}) - 0.5({x_N}^2 + \tfrac{50}{x_N}) \approx 2.34.$$

Note that in this setting $\hat{x}_N$ coincides with $x^*$ identified in (1).

### 3.2 Risk Averse Manager

We next assume the liability function can be denoted by $L(e(x_i, \upsilon), \omega) = L(x_i, \chi)$. That is, managers do not care about the source of uncertainty, i.e., whether it comes from environmental damages or the judge's decision, thus they only care about how uncertainty affects total liability. Consequently, random variables $\upsilon$ and $\omega$ are redefined as $\chi$. Manager $i$'s expected utility maximization problem is now

$$\max_{x \geq 0} U_i(x_i, \chi) = E_\chi\big[u\big(c(x_i) + L(x_i, \chi)\big)\big] \qquad (3)$$

where the operator $E_\chi$ represents the expectation with respect to the random variable $\chi$. Manager $i$ chooses the precaution level $\hat{x}_i$ that solves (3).

The following proposition considers a setting in which the judge behaves per proposition 1 but the manager is not risk neutral.

[14] This functional form has desirable well-behaved properties like costs are increasing and convex in investment while damages are decreasing and convex in investment. Additionally, risk attitude is reflected in $\rho$ where when $\rho = 0$ the manager is risk neutral, when $\rho$ is positive the manager is risk averse, and when $\rho$ is negative the manager is risk averse. The entire term is subtracted from 1000 to ensure the solution is in the first quadrant over relevant values of x.

**Proposition 2.** *If the manager is risk averse (loving) and a judge equates liability with environmental damages, then the manager over (under) invests in precaution.*

The intuition behind the above proposition is similar to that which one uses when investing in risky assets. As a risk averse investor devotes more of his wealth to a risky asset, the variance of his wealth increases. Since he is averse to this variance, he invests less in the risky asset relative to a risk neutral investor. For the case of a manager investing in precaution, as he devotes more resources into precaution, the variance of his costs decrease. Similarly, the manager is averse to this variance making him invest more in safety than a risk neutral manager. Therefore, an ex post liability rule that holds risk averse managers responsible for the full amount of damages will induce them to over-invest in safety.

**Example 2 (Risk Averse/Loving Manager).** Continuing with our previous example, suppose that all functions are the same and that the manager's utility is now represented by $\big(c(x_i) + L(x_i, \chi)\big) = 1000 - \big(c(x_i) + e(x_i, \upsilon)\big)^{\rho+1}$,[15] which is of the constant relative risk aversion form where $\rho$ is the coefficient of relative risk aversion. If $\rho = 0.5$ the manager is risk averse and the solution to the manager's problems is

$$\hat{x}_A = \underset{x_A}{\operatorname{argmax}}\, 1000 - 0.5({x_A}^2 + \tfrac{1}{x_A})^{1.5} - 0.5({x_A}^2 + \tfrac{50}{x_A})^{1.5} \approx 2.55.$$

In contrast, if the manager is risk loving $\rho = -0.5$ then the solution becomes

$$\hat{x}_L = \underset{x_L}{\operatorname{argmax}}\, 1000 - 0.5({x_L}^2 + \tfrac{1}{x_L})^{0.5} - 0.5({x_L}^2 + \tfrac{50}{x_L})^{0.5} \approx 1.93.$$

Hence, our example shows that the risk averse manager overinvests in safety while the risk-loving manager underinvests compared to the risk neutral manager, i.e. $\hat{x}_A > \hat{x}_N > \hat{x}_L$.

[15] Since the judge equates liability with damages $\chi$ coincides with $\upsilon$.

### 3.3 Ambiguity Averse Manager

An ambiguity averse manager behaves differently than a manager that is only risk averse, since ambiguity aversion is an aversion to unknown risks, that is, he exhibits a preference for known risks relative to unknown risks. Following Klibanoff (2005), an ambiguity averse manager evaluates disutility with an increasing and concave function $\phi(\cdot)$, where $\frac{\partial \phi}{\partial u} > 0$ and $\frac{\partial^2 \phi}{\partial u^2} < 0$. That is, $\phi(\cdot)$ is a monotonic transformation of $u(\cdot)$ and ambiguity averse managers prefer situations where outcomes are more certain. This specification can be understood as an aversion to mean preserving spreads in the distribution of expected utilities. Additionally, it is assumed that the random variable $\chi$ is distributed by $F_\chi(\sigma)$ where $\sigma$ is a random variable which is distributed by $F_\sigma(\pi)$. That is, $\pi$ is a parameter that defines the distribution of the random variable $\sigma$, which in turn defines the distribution of the random variable $\chi$. In other words, there is a distribution of distributions of the variable $\chi$. Let us represent the expected utility maximization problem of an ambiguity averse manager $i$ as follows,

$$\max_{x \geq 0} \Phi_i(x_i; \chi) = E_\sigma\left[\phi_i\left(E_\chi\left[u_i\left(c(x_i) + L(x_i, \chi)\right)\right]\right)\right], \qquad (4)$$

hence, the manager's optimal precaution level is $\hat{x}_i$, which solves (4).

**Proposition 3.** *An ambiguity averse manager invests more in precaution relative to their risk averse or risk loving counterpart, ceteris paribus.*

An ambiguity averse manager avoids ambiguity analogously to how a risk averse manager avoids risk. A risk averse manager avoids risk, i.e. variance, by investing more in safety which reduces the variance of his expected costs. In the case of an ambiguity averse manager, he avoids ambiguity by investing even more. This occurs since the manager tends to favor the more pessimistic probability distribution for damages. Our results provide a clear distinction between the way a risk averse and ambiguity averse manager responds to uncertainty. Therefore, a liability rule that equates liability with damages will not induce managers to invest optimally in precaution. Holding managers that are risk averse and ambiguity averse liable for the full damages causes them to over-invest, and such behavior is emphasized by ambiguity. Therefore, reducing liability below the suggested optimal rule could induce them to invest the optimal amount in precaution, as shown in example 3.

**Example 3 (Ambiguity Averse Manager).** Suppose that all functions are the same as in the previous examples, but we now consider the following monotonic transformation $\phi = -\big(u(\cdot)\big)^{-(\mu+1)}$ that can be expressed as

$$\phi_i\big(E_\chi\big[u_i\big(c(x_i) + L(x_i,\chi)\big)\big]\big) = -(1000 - (c(x_i) + L(x_i,\chi))^{\rho+1})^{-(\mu+1)} \text{ ,}$$

which is of the constant relative ambiguity aversion form where $\mu$ is the coefficient of relative ambiguity aversion. In addition, since $L(x_i,\chi) = e(x_i,\upsilon)$, then $\chi = \nu$. If $\mu = 9$ and $\rho = 0.5$ (which was assumed for the case of the risk averse manager), the solution to the ambiguity and risk averse manager's problems is

$$\hat{x}_{AA} = \underset{x_{AA}}{\operatorname{argmax}} -0.5\left(1000 - 0.01({x_{AA}}^2 + \frac{1}{x_{AA}})^{1.5} - 0.99({x_{AA}}^2 + \frac{50}{x_{AA}})^{1.5}\right)^{-10}$$
$$- 0.5\left(1000 - 0.99({x_{AA}}^2 + \frac{1}{x_{AA}})^{1.5} - 0.01({x_{AA}}^2 + \frac{50}{x_{AA}})^{1.5}\right)^{-10} \approx 2.78,$$

where AA denotes ambiguity averse. However, if $\rho = -0.5$ then the solution to the ambiguity averse but risk loving manager's problems is

$$\hat{x}_{AL} = \underset{x_{AL}}{\operatorname{argmax}} -0.5\left(1000 - 0.01({x_{AL}}^2 + \frac{1}{x_{AL}})^{0.5} - 0.99({x_{AL}}^2 + \frac{50}{x_{AL}})^{0.5}\right)^{-10}$$
$$- 0.5\left(1000 - 0.99({x_{AL}}^2 + \frac{1}{x_{AL}})^{0.5} - 0.01({x_{AL}}^2 + \frac{50}{x_{AL}})^{0.5}\right)^{-10} \approx 1.96.$$

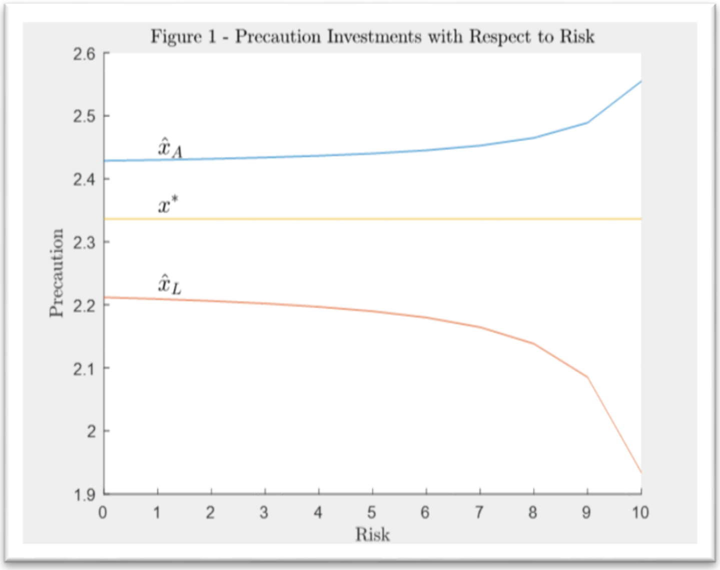
Hence, the results of our example indicate significant overinvestment for the ambiguity averse manager who is also risk averse, $\hat{x}_{AA} > \hat{x}_A$. However, when the manager is ambiguity averse and risk loving the underinvestment (compare to the risk neutral) is less severe than the case in which ambiguity is absent, i.e. $\hat{x}_{AL} > \hat{x}_L$.

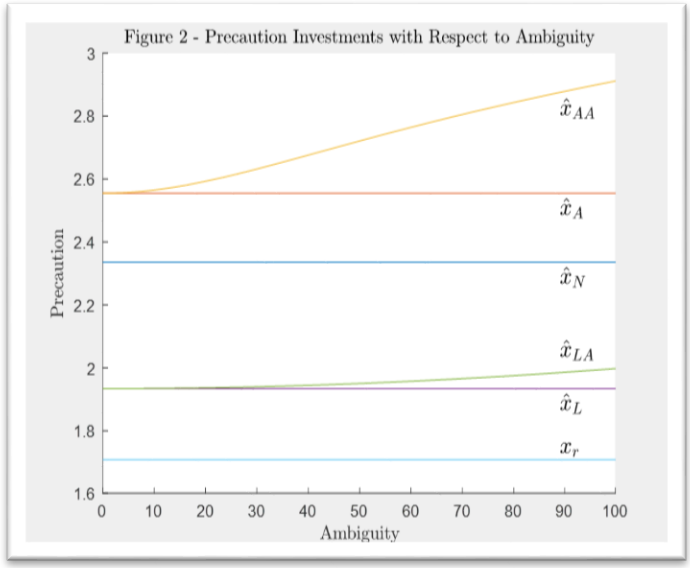
## 4. Discussion

### 4.1 Uncertainty

These results stand in contrast to those in Kolstad et al. (1990) where increased uncertainty results in a reduction in precaution. Our results show that if a manager is ambiguity averse (risk averse) increasing ambiguity (risk, respectively) leads to an increase in precaution. The difference in results stems from the different definition of uncertainty used here compared to the one used in Kolstad et al. (1990). They assume that managers are risk neutral and increasing uncertainty is defined by introducing a mean preserving spread of the probability of being found liable. In effect, it causes expected liability to go down and it logically follows that investment in precaution would decrease. Here, however, it is not assumed managers are only risk neutral and increasing uncertainty is defined by holding expected liability constant while increasing risk or ambiguity with respect to liability. Increasing risk causes risk loving managers to invest less and risk averse manager to invest more. If managers are ambiguity averse they increase precaution when ambiguity increases regardless if they are risk averse or risk loving. These results are summarized in figures 1 and 2.[16] For a given degree of ambiguity aversion, an increase in risk aversion also increases investment. Whether uncertainty increases or decreases investment in precaution has large implications when comparing ex ante regulation vs. ex post liability.

[16] The different scales in figures 1 and 2 are for easy readability. For example, if figure 1 were scaled from 0 to 100 the changes in investment levels would not be easy to observe. Also, the starting precaution levels for the risk averse and risk loving types are not the same since figure 1 begins with minimal risk and figure 2 begins with maximal risk. Note the precaution level at the top of the risk index is the same as that of figure 2.




If more uncertainty reduces investments in precaution under ex post liability then ex ante regulation would tend to be viewed as more favorable in situations with more uncertainty. In this case, when relying on only tort liability to induce firms to invest in precaution, a regulator may reach the conclusion that liability should be as clear as possible. That is, any uncertainty which can be eliminated should be. On the other hand, if it is assumed that more uncertainty increases investment in precaution then ex ante regulation would tend to be viewed as less efficient with more uncertainty as long as it does not lead to over-investment. However, whether ex ante regulation or ex post liability is more efficient also depends on other factors. . For example, consider a risk loving manager facing high expected liability, $\chi = 70$ in the event an accident occurs, and as in previous examples damages are such that $\nu = 50$. Also, regulation enforcement costs are such that $c(x) + r(x) = x^2 + x^{0.6}$.[17] If risk is low, high liability induces the manager to over-invest in precaution (figure 3 depicts such a scenario). It is less clear whether liability or regulation is more efficient. Both are inefficient such that regulation leads to under-investment and

[17] These damage and cost levels make it such that the efficient precaution level is $x^* = 2.3362$ and the efficient ex ante regulation precaution level is $x_r = 2.2663$.

liability leads to over-investment. If we assume symmetry with respect to efficiency then the simple measure of the absolute value of the difference in the investments and the optimal level for both policies is sufficient.

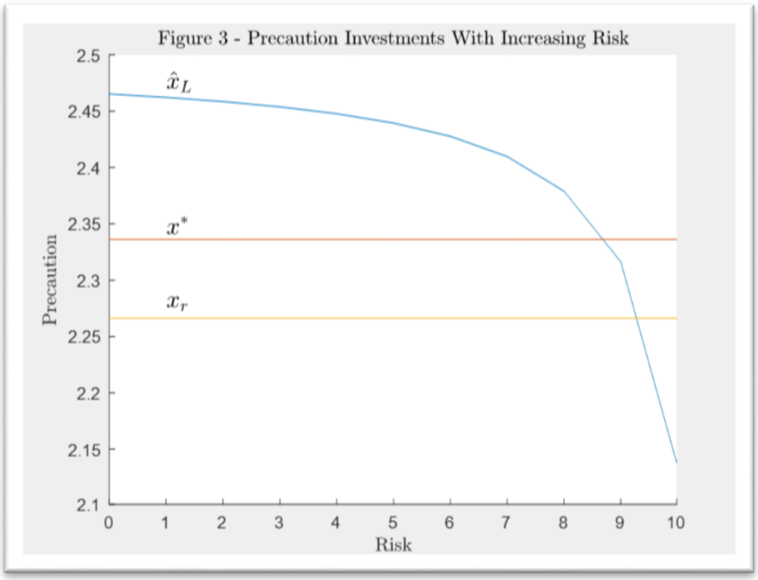



Figure 3 above also shows that at low levels of risk the difference in investment for ex-post liability is greater than that under ex-ante regulation. By the simple measure regulation is more efficient. But, as risk increases for the risk loving manager, he begins to invest less until the difference for liability becomes smaller than regulation while they are still over-investing making liability more efficient. As risk continues to increase investment under liability turns to under-investment yet is still more efficient until it crosses below the ex ante level making regulation more efficient again. The story is similar for a risk averse or ambiguity averse manager facing low expected liability.

### 4.2 Uncertainty Attitudes

Suppose we observe a manager over-investing in precaution when expected liability equals expected damages where $\hat{x} = 2.55$, but we cannot observe what his uncertainty attitude is. We can argue that there are different attitudes that result in a manager investing such an amount. The purely risk averse manager (from example 2, with a risk aversion parameter $\rho = 0.5$) invests such an amount. Another manager that is less risk averse, where $\rho = 0.25$, and ambiguity averse where $\mu = 5$, also invests $\hat{x} = 2.55$ as does a manager that is minimally risk averse, where $\rho = 0.01$, but very ambiguity averse, where $\mu = 31.5$. If it is assumed the manager is the purely risk averse type and it is desired to induce them to invest the optimal amount in precaution by lowering expected liability, then setting $\chi_h$ equal to 38.24 causes him to invest the optimal amount.

$$\hat{x}_A = \underset{x_A}{\operatorname{argmax}}\, 1000 - 0.5({x_i}^2 + \frac{1}{x_i})^{1.5} - 0.5({x_i}^2 + \frac{38.24}{x_i})^{1.5} \approx 2.34.$$

But, if it is not the case that the manager is only risk averse, and he is ambiguity averse such that $\rho = 0.01$ and $\mu = 31.5$ then setting $\chi_h$ equal to 38.24 causes him to under-invest in precaution where $\hat{x} = 2.31$. That is,

$$\hat{x}_{AA} = \underset{x_{AA}}{\operatorname{argmax}} -0.5\left(1000 - 0.01({x_i}^2 + \frac{1}{x_i})^{1.07} - 0.99({x_i}^2 + \frac{38.24}{x_i})^{1.07}\right)^{-20}$$
$$- 0.5\left(1000 - 0.99({x_i}^2 + \frac{1}{x_i})^{1.07} - 0.01({x_i}^2 + \frac{38.24}{x_i})^{1.07}\right)^{-20} \approx 2.31.$$

Whether a given liability level induces optimal investment in precaution is highly sensitive to manager's attitudes toward uncertainty. In the example depicted above two different managers who invest the same amount above the optimal level when liability equals damages do not invest the same amount when liability is reduced. This is important for policy because it demonstrates that incorrect assumptions about manager behavior can lead to inefficient policy choices. It also shows that the distinction between risk aversion and ambiguity aversion matters. As shown in Kunreuther et al. (1995), agents can display these attitudes and if policymakers do not account for the fact that behavior varies between objective and subjective probabilities when choosing policy, it may cause them to choose incorrect policy prescriptions.

### 4.3 Heterogeneity

Shavell (1984) assumes that external damages caused by firms are heterogeneous. Therefore, the optimal ex ante regulation standard equals the level of care that would be efficient for a firm that causes average external damages. However, such a policy is inefficient since any firms that do not cause average external damages will over or under invest in precaution. Similarly, we find that heterogeneity of manager's attitudes toward uncertainty renders ex post liability inefficient. If expected liability is the same for all managers, equal to expected damages, and managers have different attitudes toward uncertainty, then only a risk and ambiguity neutral manager will invest in the optimal level of precaution. Any manager that is not risk neutral will over or under invest. For example, suppose that all managers are risk averse and some are ambiguity averse. These managers will over-invest in precaution if expected liability is equated with expected damages. Less investment can be induced by decreasing expected liability. Since all firms are overinvesting similarly due to their attitudes toward uncertainty it is possible to decrease liability such that all invest in precaution at a level that is closer to the level which is optimal. However, if managers are heterogenous as depicted in figure 2 then decreasing expected liability pushes risk loving managers to under-invest even more while risk averse managers reduce their over-investment. In this case there is no way to bring all firms closer to the optimal amount. Increasing or decreasing liability always makes the investment of one type more inefficient.

## 5. Conclusion

Our results indicate that increasing uncertainty reduces precaution only when managers are risk loving. On the contrary, when managers are risk averse and ambiguity averse an increase in risk or ambiguity increases precaution. When managers are risk neutral and ambiguity neutral changes in uncertainty have no effect on precaution. Therefore, if expected liability equals expected damages risk neutral managers will invest the optimal amount in precaution while risk averse manager over-invest and risk loving managers under-invest. All else equal, ambiguity averse managers invest more in precaution. Punitive expected liability can induce risk loving managers to invest the optimal amount while expected liability that is less than expected damages induces risk averse managers to invest the optimal amount. Thus, given sufficiently high heterogeneity of manager's attitudes toward uncertainty ex ante regulation is more efficient than ex post liability. Whether ex ante regulation or ex post liability is more efficient greatly depends on uncertainty attitudes. Hence, the design of environmental regulation needs to be reevaluated in contexts where ambiguity is present. Future research can extend this framework by introducing an option for policymakers to acquire more information about the managers' attitude towards uncertainty. In this context, the regulator needs to balance the cost of acquiring information and, thus, reducing uncertainty, with the benefits from reaching an optimal precaution level.

# Appendix

***Proof of Proposition 1***

If the manager's utility is represented by $u[c(x_i) + L(e(x_i, \upsilon), \omega)] = c(x_i) + L(e(x_i, \upsilon), \omega)$ then setting $L(e(x_i, \upsilon), \omega) = e(x_i, \upsilon)$ induces the manager to choose $x_i^*$ that minimizes costs. Risk neutral manager $i$'s optimization problem is

$$\underset{x_i}{\operatorname{argmin}}\, c(x_i) + E_\upsilon[e(x_i, \upsilon)].$$

The first order condition is

$$c'(x_i) = -E_\upsilon[e'(x_i, \upsilon)].$$

This condition equates the marginal cost of safety with the expected marginal benefit which is the negative marginal environmental damages. This is the same condition as the social planner's problem and therefore minimizes social costs associated with safety.

***Proof of Proposition 2***

Expected utility for risk averse manager $i$ is

$$U_i(x_i, \chi) = E_\chi\big[u_i\big(c(x_i) + L(s_i, \chi)\big)\big].$$

The first order condition is

$$\gamma_i(x_i^*) \equiv E_\chi\big[u'_i\big(c(x_i) + L(x_i, \chi)\big)\big(c'(x_i) + L'(x_i, \chi)\big)\big] = 0.$$

We can differentiate $\gamma_i(x_i^*)$ to obtain

$$E_\chi\left[u''_i(\cdot)\big(c''(x_i) + L''(x_i, \chi)\big)^2 + u'_i(\cdot)\big(c''(x_i) + L''(x_i, \chi)\big)\right] < 0.$$

Since $u' < 0,\ u'' < 0$ by concavity of the utility function and $c''(x_i) + L''(x_i, \chi) > 0$ by convexity of the cost and damage functions. Hence, $\gamma'_i(x_i^*)$ is decreasing in $x_i$, entailing that $\gamma_i(x_j) < 0$ for all $x_j > x_i^*$ but $\gamma_i(x_j) > 0$ otherwise. Define the utility function of manager 2 as more risk averse than manager 1, that is, $u_2(\cdot) = \psi[u_1(\cdot)]$ where $\psi(\cdot)$ is an increasing convex transformation. Thus, the first order conditions for managers 1 and 2 are

$$\gamma_1(x_1^*) \equiv E_\chi\left[u'_1(\cdot)\big(c'(x_1) + L'(x_1, \chi)\big)\right] = 0 \qquad (1')$$

$$\gamma_2(x_2^*) \equiv E_\chi\left[\psi'\big(u_1(\cdot)\big)u'_1(\cdot)\big(c'(x_2) + L'(x_2, \chi)\big)\right] = 0.$$

If we evaluate $\gamma_2(\cdot)$ at $x_1^*$ we find that

$$\gamma_2(x_1^*) \equiv E_\chi\left[\psi'\big(u_1(\cdot)\big)u'_1(\cdot)\big(c'(x_1) + L'(x_1, \chi)\big)\right] > 0. \qquad (2')$$

The expression in (2') is the same as the expression in (1') except that each term is multiplied by $\psi'(u_1(\cdot))$. Since this term is a positive decreasing function of $\chi$, the expression (2') over-weights lower values of $\chi$ relative to lower values, this increases (2') relative to (1') since the lower values of $\chi$ are more preferred outcomes they have a positive expected value. Therefore, (2') sums to a value greater than zero. To show $\psi'(u_1(c(x_i) + L(x_i, \chi)))$ is increasing in $\chi$ we can apply the Envelope Theorem and differentiate $\psi'(\cdot)$ with respect to $\chi$ to obtain

$$\psi''\Big(u_1\big(c(x_1^*) + L(x_1^*, \chi)\big)\Big)\,u'_1\big(c(x_1^*) + L(x_1^*, \chi)\big)\frac{\partial L}{\partial \chi}(x_1^*) > 0.$$

Since $\psi'' > 0$ by convexity of the transformation $\psi$, $u' < 0$ since utility is decreasing in total costs, and $\frac{\partial L}{\partial \chi} > 0$ since liability is increasing in $\chi$. Thus, $x_1^* < x_2^*$ , a more risk averse decision maker within a firm invests more in safety.

***Proof of Proposition 3***

Consider the random variable $\chi$ distributed according to $F_\chi(\sigma)$ where $\sigma$ is distributed according to $F_\sigma(\pi)$. Expected utility for ambiguity averse manager $i$ is

$$\Phi_i(x_i;\chi) = E_\sigma E_\chi\big[\phi_i\big(\big[u_i\big(c(x_i) + L(s_i,\chi)\big)\big]\big)\big].$$

The first order condition yields

$$\theta_i(x_i^*) \equiv E_\sigma E_\chi\big[\phi'_i\big(\big[u_i\big(c(x_i) + L(x_i,\chi)\big)\big]\big)\big]\big(\big[u'_i\big(c(x_i) + L(x_i,\chi)\big)\big(c'(x_i) + L'(x_i,\chi)\big)\big]\big) = 0.$$

We can differentiate $\theta_i(x_i^*)$ to obtain

$$\theta'_i(x_i^*) = E_\sigma E_\chi\Big[\phi''_i(\cdot)u'_i(\cdot)^2\big(c'(x_i) + L'(x_i,\chi)\big)^2 + \phi'_i(\cdot)u''_i(\cdot)\big(c'(x_i) + L'(x_i,\chi)\big)^2$$

$$+ \phi'_i(\cdot)u'_i(\cdot)\big(c''(x_i) + L''(x_i,\chi)\big)\Big] < 0.$$

Since $\phi'' < 0$ by concavity of the ambiguity attitude function, $u'' < 0$ by concavity of the utility function and $c'' + L'' > 0$ by convexity of the cost and damage function. Hence, $\gamma'_i(x_i^*)$ is decreasing in $x_i$, entailing that $\gamma_i(x_j) < 0$ for all $x_j > x_i^*$ but $\gamma_i(x_j) > 0$ otherwise. Define the ambiguity function of manager 2 as more ambiguity averse than manager 1, that is, $\phi_2(x) = \xi(\phi_1(x))$ where $\xi(\cdot)$ is an increasing convex transformation. Thus, the first order conditions for managers 1 and 2 are

$$\theta_1(x_1^*) \equiv E_\sigma\big[\phi'_1(E_v[u_1(\cdot)])\big]\big(E_\chi\big[u'_1(\cdot)\big(c'(x_1) + L'(x_1,\chi)\big)\big]\big) = 0 \qquad (3')$$

$$\theta_2(x_2^*) \equiv E_\sigma\Big[\xi'\Big(\phi_2\big(E_\chi[u_1(\cdot)]\big)\Big)\phi'_i\big(E_\chi[u_1(\cdot)]\big)\Big]\big(E_\chi\big[u'_1(\cdot)\big(c'(x_2) + L'(x_2,\chi)\big)\big]\big) = 0.$$

If we evaluate $\theta_2(\cdot)$ at $x_1^*$ derived from (3), we find that

$$\theta_2(x_1^*) \equiv E_\sigma\Big[\xi'\Big(\phi_2\big(E_\chi[u_2(\cdot)]\big)\Big)\phi'_2\big(E_\chi[u_2(\cdot)]\big)\Big]\big(E_\chi\big[u'_2(\cdot)\big(c'(x_2) + L'(x_2,\chi)\big)\big]\big) > 0\,. \qquad (4')$$

Notice, (4') is the same as (3'), which is equal to zero, except that it is multiplied by $\xi'(\cdot)$. Since this term is a positive decreasing function of $\chi$, the summation (4') over-weights lowers values of $\chi$ relative to higher values, this increases (4') relative to (3'), and therefore (4') sums to a value greater than zero. To show $\xi'(\cdot)$ is increasing in $\chi$ we can apply the Envelope Theorem and differentiate $\xi'(\cdot)$ with respect to $\chi$ to obtain

$$\xi''\left(\phi_2\left(E_\chi[u_2(\cdot)]\right)\right)\phi'_2\left(E_\chi[u_2(\cdot)]\right)\left(E_\chi\left[u'_2(\cdot)\frac{\partial L}{\partial \chi}(x_1^*)\right]\right) < 0.$$

Since $\xi'' > 0$ by convexity of the transformation $\gamma$, $\phi' > 0$ by the ambiguity attitude function increasing in utility, $u' < 0$ by utility decreasing in total cost, and $\frac{\partial L}{\partial \chi} > 0$ since total cost is increasing in $\chi$. Thus, $x_1^* < x_2^*$ , a more ambiguity averse decision maker within a firm invests more in safety.

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