A Solution to the Problem of Externalities When Agents Are Well-Informed

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Introduction

- There is a unilateral externality
- The agents involved know the relevant technology and the tastes of all other agents.
- The **"regulator"** who has the responsibility for determining the final allocation, does not have this information

- How can the regulator design a mechanism that will implement an efficient allocation?

Introduction

- Simple two-stage games whose subgame-perfect equilibria implement efficient allocations
  - In the case of public goods, the mechanisms implement Lindahl allocations;
  - in the case of a negative externality, the injured parties are compensated (*compensation mechanisms*)

- Two agents
- Firm 1 produces output x
    - $\max \pi_1 = rx - c(x)$
- Firm 2's profits: $\pi_2 = -e(x)$ [negative externality!]
- All of this information is known to both agents but is not known by the regulator
- x will not be efficient

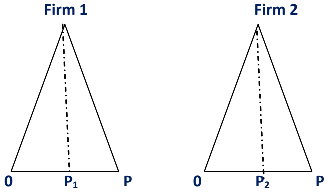
- There are three classic solutions to this problem of externalities:

1. Ronald Coase (1960), involves negotiation between the agents.
2. Kenneth Arrow (1970), involves setting up a market for the externality.
3. C. Pigou (1920), involves intervention by a regulator who imposes a Pigovian tax

- Assume that the government has full information, then,
  - Internalizing the externality..... EASY!
  - $\max_x rx - c(x) - e(x)$
  - Pigovian tax: $P^* = e'(x^*)$
  - $\max_x rx - c(x) - P^* x$

- However, the regulator does not know the externality cost function and cannot determine the appropriate value of p*

- Design a mechanism that induces the agents to reveal their information and achieve an efficient level of production.
- Compensation mechanism that solves the regulator's problem
  - **Announcement stage**. Firm 1 and 2 simultaneously announce the magnitude of the appropriate Pigovian tax; p_1 and p_2.
  - **Choice stage**. The regulator makes side-payments to the firms. The two firms face profit-maximization problems:
  - $\pi_1 = rx - c(x) - p_2 x - \alpha_1 (p_1 - p_2)^2$
  - $\pi_2 = p_1 x - e(x)$



Announcement Stage

$\max \Pi_1 \qquad \max \quad \Pi_2$

Choice Stage

- Unique SPE of this game:
  - $p_1 = p_2 = p^*$ and x^*
- Backward Induction, Firm 1 maximizes its profits,
  - $r = c'(x) + p_2$
  - This determines the optimal choice, x, as $x(p_2)$. Note that $x'(p2) < 0$
- Price-setting stage of the game:
  - If firm 1 believes that firm 2 will announce p_2, then: $p_1 = p_2$

- Firm 2's pricing decision.
- Although firm 2's announcement:
  - No direct effect on firm 2's profits,
  - Indirect effect through the influence of p_2 on firm 1's output choice in stage 2
  - $\pi_2'(p_2) = [p_1 - e'(x)]x'(p_2) = 0$
  - $p_1 = e'(x)$, therefore
  - $r = c'(x) + e'(x)$ which is the condition for social optimality!

- For example,suppose that firm 1 thinks that firm 2 will report a large price for the externality..
- Then, since firm 1 is penalized if it announces something different from firm 2
- Firm 1 will also want to announce a large price
- If firm 1 announces a large price, firm 2 will be "*overcompensated*" for the externality
- But firm 2 can give firm 1 an incentive to produce a large amount of output iff it reports a **small** price for the externality
- This contradicts the original assumption
- The only equilibrium for the mechanisms occurs if firm 2 is just compensated (*on the margin*) for the cost that firm 1 imposes on it

Three agents

- Suppose that agent 1 imposes an externality on agents 2 and 3.

- p_{ij}^{k} represents the price announced by *agent* k that measures (in equilibrium) the marginal cost that agent j's choice imposes on agent i.
- Compensation mechanism for this problem has payments of the form:
- $\pi_1 = rx - c(x) - [p_{21}^{2} + p_{31}^{3}]x - \left\| p_{21}^{1} - p_{21}^{2} \right\| - \left\| p_{31}^{1} - p_{31}^{3} \right\|$
- $\pi_2 = p_{21}^{1} - e_2(x)$
- $\pi_3 = p_{31}^{1} - e_3(x)$

- If payments are distributed so as to balance the budget out of equilibrium, the payoffs become,
- $\pi_1 = rx - c(x) - [p_{21}^2 + p_{31}^3]x - \|p_{21}^1 - p_{21}^2\| - \|p_{31}^1 - p_{31}^3\|$
- $\pi_2 = p_{21}^1 x - e_2(x) + \|p_{31}^3 - p_{31}^1\| x + \|p_{31}^1 - p_{31}^3\|$
- $\pi_3 = p_{31}^1 x - e_3(x) + \|p_{21}^2 - p_{21}^1\| x + \|p_{21}^1 - p_{21}^2\|$

- it is possible to verify that the unique equilibrium of this mechanism is the efficient outcome.
- In fact, it is not necessary to have penalty terms *when there are more* than two agents
  - set the penalty terms above equal to zero
  - $r - c'(x) - [p_{21}^2 + p_{31}^3] = 0$
  - $[p_{21}^1 - e_2'(x) + p_{31}^3 - p_{31}^1]x'(p_{21}^2 + p_{31}^3) = 0$
  - $[p_{31}^1 - e_3'(x) + p_{21}^2 - p_{21}^1]x'(p_{21}^2 + p_{31}^3) = 0.$
- The equilibrium is efficient

- Linear prices are fine in a convex environment, but if the environment is not convex, linear prices will not support efficient allocations.
- Generalization of the CM:
  - **Announcement stage**. Firm 1 and 2 each announce the externality costy function for firm 2: $e_1(\cdot)$.and $e_2(\cdot)$
  - **Choice stage**. Firm 1 chooses x and each firm receives payoffs given by:
  - $\pi_1 = rx - c(x) - e_2(x) - \|e_1 - e_2\|$
  - $\pi_2 = e_1(x) - e(x)$
  - Note that $\|e_1 - e_2\|$ represents any norm in the appropriate function space.

- In equilibrium firm 1 will always want to report the same function as firm 2: $e_1(x) \equiv e_2(x)$
- Maximization of profits by firm 1 implies

$$rx^* - c(x^*) - e_2(x^*) \geq rx - c(x) - e_2(x) \qquad ((1))$$

- The equilibrium choice of x must also *max*. firm 2's profits:

$$e_1(x^*) - e(x^*) \geq e_1(x) - e(x) \qquad ((2))$$

- adding (1) and (2) and using $e_1(x) \equiv e_2(x)$

$$rx^* - c(x^*) - e(x^*) \geq rx - c(x) - e(x)$$

- which shows that x^* is the socially optimal amount

- *Analyze the case of a repeated game and non-convex environment!!*

- The compensation mechanism provides a simple mechanism for internalizing externalities in economic environment
- Transfer payments can be chosen so that the compensation mechanism is balanced, and penalty payments, when they are used, can be chosen to be arbitrarily small.
- The main problem with the mechanism is that it re- quires complete information by the agent