MSNE and Simultaneous Move Games of Incomplete Information

Mixed-strategy Nash equilibrium

- Players could randomize (mix) their choices, e.g., choosing strategy A with probability p and strategy B with probability $1-p$. Allowing for the strategy of every player i to be defined as a probability distribution σ_i over the strategy space S_i, we can extend the definition of NE to the following mixed strategy Nash Equilibrium (msNE):

Definition msNE

- **Definition of msNE**: *Consider a strategy profile* $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ *where* σ_i *is a mixed strategy for player* i. *Strategy profile* σ *is a msNE if and only if*

$$\pi_i(\sigma_i, \sigma_{-i}) \geq \pi_i\left(s_i^{'}, \sigma_{-i}\right) for\ all\ s_i^{'} \in S_i\ and\ for\ all\ i$$

That is, σ_i *is a best response of player* i *to the strategy profile* σ_{-i} *of the other* $N - 1$ *players, i.e.,* $\sigma_i = BR_i(\sigma_{-i})$.

Important Point

- In a msNE players must be indifferent among all (or at least some) of their pure strategies. That is, if a player randomizes over all his pure strategies (or over a subset of them), then he must receive the same expected payoff from each of those pure strategies. Otherwise, he would prefer one strategy over another, and thus choose such strategy with 100% probability, i.e., play a pure strategy.

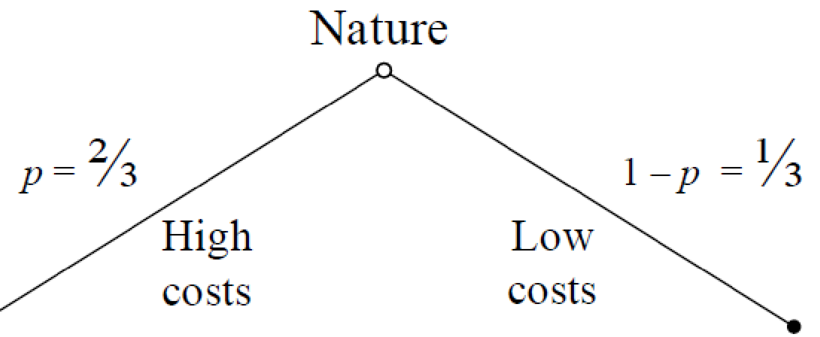
Simultaneous Move Games of Incomplete Information

- Players act under "incomplete information" if at least one player cannot observe a piece of information.

- We often refer to this piece of private information as player i's "type" and denote it as θ_i. While player j might not observe player i's type (e.g., its marginal costs), he knows the probability distribution of each type.

- For instance, if marginal costs can be either high or low, i.e., $\Theta_i = \{H, L\}$, the probability of firm i's costs being high is $p(\theta_i = H) = p$ whereas the probability of its costs being low is $p(\theta_i = L) = 1 - p$ where $p \in (0,1)$. The literature refers to these incomplete information games in which players act simultaneously as *Bayesian games*.

Example



High costs:

Firm A \ Firm B	Invest	Not Invest
Invest	6, 1	3, 0
Not Invest	0, 0	1, 3

Low costs:

Firm A \ Firm B	Invest	Not Invest
Invest	3, 1	0, 0
Not Invest	0, 0	1, 3

Bayesian games

- Note that every player i's strategy in an incomplete information context needs to be a function of its privately observed type θ_i (e.g., its benefit from investing in a new technology), implying that s_i is a function $s_i(\theta_i)$. Importantly, player i's strategy is not conditioned on other players' types,

$$\theta_{-i} = (\theta_1, \theta_2, \dots, \theta_{i-1}, \theta_{i+1}, \dots \theta_n).$$

- That is, we do not write $s_i(\theta_i, \theta_{-i})$ since player i cannot observe the types of all other players. (In fact, if all players could observe the types of all of their rivals, we would be describing a complete information game, as those in previous sections of the chapter.)

- For simplicity, we assume that types are independently distributed, which entails that every player i cannot infer additional information about his rivals' types θ_{-i} after observing his own type θ_i.

Bayesian Nash Equilibrium

- *A strategy profile* $\left(s_1^*(\theta_1), s_2^*(\theta_2), \ldots, s_n^*(\theta_n)\right)$ *is a Bayesian Nash Equilibrium, BNE, of a game of incomplete information if*

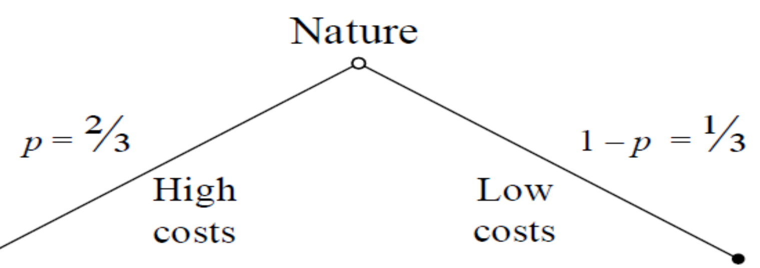
$$EU_i(s_i^*(\theta_i), s_{-i}^*(\theta_{-i}); \theta_i, \theta_{-i}) \geq EU_i(s_i(\theta_i), s_{-i}^*(\theta_{-i}); \theta_i, \theta_{-i})$$

for every strategy $s_i(\theta_i) \in S_i$, *every type* $\theta_i \in \Theta_i$, *and every player* i.

- In words, when all other players select equilibrium strategies, the expected utility that player i obtains from selecting $s_i^*(\theta_i)$ when his type is θ_i is larger than that of deviating to any other strategy $s_i(\theta_i)$.

Example

- Let us next apply this solution concept to the incomplete information version of the technology adoption game: (1) Identify the strategy set for each player



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Bayesian-normal form representation

We now need to find the *expected payoffs* that would go in every cell.

Firm B

Firm A		I	NI
	$I_H I_L$		
	$I_H NI_L$		
	$NI_H I_L$		
	$NI_H NI_L$		

Firm *B* (columns), Firm *A* (rows)

	I	*NI*
$I_H I_L$	5, 1	2, 0
$I_H NI_L$	4, ⅔	2 ⅓, 1
$NI_H I_L$	1, ⅓	⅔, 2
$NI_H NI_L$	0, 0	1, 3

Firm *B* (columns), Firm *A* (rows)

	I	*NI*
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Alternative approach

- Parameter α represents the probability that Firm *A* invests when it is of the High type, γ is the probability that firm *A* invests when its type is Low, and β represents the probability that the uninformed Firm *B* invests.

Firm *A*		Firm *B*: Invest (β)	Firm *B*: Not Invest ($1-\beta$)
Invest	α	6, 1	3, 0
Not Invest	$1-\alpha$	0, 0	1, 3

Firm A is High type with probability p

Firm *A*		Firm *B*: Invest (β)	Firm *B*: Not Invest ($1-\beta$)
Invest	γ	3, 1	0, 0
Not Invest	$1-\gamma$	0, 0	1, 3

Firm A is Low type with probability $1-p$