Externalities and PG

MWG- Chapter 11

Simple Bilateral Externality

- *When external effects are present, CE are not PO. Assume:*

1. Two consumers $i = 1, 2$
2. The actions of these consumers do not affect prices $p \in \mathbb{R}^L$
3. w_i Consumers i's wealth
4. $U_i(x_{1i}, ..., x_{Li}, h)$
5. $\frac{\partial U_2}{\partial h} \neq 0$, consumer 1's choice of h affects consumer 2's well-being (externality)

- **Each consumer i derived utility function over the level of h:**

$$\begin{aligned} v_i(p, w_i, h) &= \max_{x_i \geq 0} u_i(x_i, h) \\ s.t \quad p \times x_i &\leq w_i \end{aligned}$$

- We shall assume that the consumer's ut. function takes a *quasilinear* form:
- $v_i(\cdot) = \phi_i(p, h) + w_i$
- we can rewrite $\phi_i(h)$ and assume $\phi_i''(\cdot) < 0$

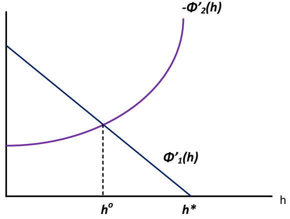
- Competitive Equilibrium: *each of the two consumers maximize her utility limited only by her wealth and P*

$$\begin{aligned} \phi_1'(h^*) &\leq 0, \text{ with equality if } h^* > 0 \\ \text{Interior solution} &: \phi_1'(h^*) = 0 \end{aligned}$$

- Pareto Optimal Allocation: *the optimal level of h must maximize the JOIN surplus of the 2 consumers*

$$\begin{aligned} & \max \phi_1(h) + \phi_2(h) \\ FOC &: \phi_1'(h^o) \leq -\phi_2'(h^o) \\ \text{Interior solution} &: \phi_1'(h^o) = -\phi_2'(h^o) \end{aligned}$$

- Considers $(h^o, h^*) >> 0$. If $\phi_2'(\cdot) < 0$ (so h generates negative ext). Then, we have $\phi_1'(h^o) = -\phi_2'(h^o) > 0$, because $\phi_1'(h^o)$ is decreasing and $\phi_1'(h^o) = 0 \rightarrow h^o < h^*$



Quotas and Taxes

- Suppose negative externality $h^o < h^*$

1. Government intervention to achieve efficiency is the direct control of the externality: Mandate $h = h^o$
2. Tax on the externality-generating activity

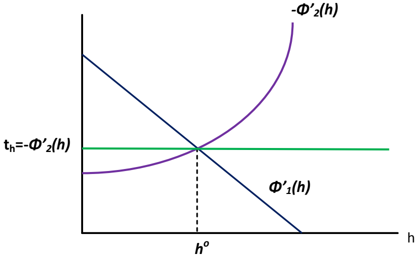
- Pigouvian Tax: $(A)\ t_h = -\phi_2'(h^o) > 0$
  - Consumer 1 then choose the level of h that solves:

$$\max_{h \geq 0} \phi_1(h) - t_h \times h$$

$$(B)\ FOC \quad : \quad \phi_1'(h) \leq t_h \ \ \text{(with equality if } h > 0\text{)}$$

  - Given that: $t_h = -\phi_2'(h^o)$, $h = h^o$ satisfies (B). In addition, $\phi_1''(h) < 0$, h^o must be unique to solution (A)

Quotas and Taxes

- 

- Assume: Property Rights with regard to the externality-generating activity
  - Assign the right to an externality-free environment to consumer 2
  - Consumer 1 is unable to produce externality without Consumer 2's permission
  - Assume consumer 2 makes consumer 1 a take-it-or-leave-it offer demanding a payment T
  - Consumer 1 accepts iff $\phi_1(h) - T \geq \phi_1(0)$
- Consumer 2 will choose her offer (h, T) to solve

$$\begin{aligned}
& \underset{h\geq 0, T}{Max}\, \phi_2(h) + T \\
s.t\ \phi_1(h) - T &\geq \phi_1(0) \\
& \underset{h\geq 0}{Max}\ \phi_2(h) + \phi_1(h) - \phi_1(0) \\
FOC &: \phi_2'(h) + \phi_1'(h) = 0 \\
h^o &= \phi_1'(h) = -\phi_2'(h)
\end{aligned}$$

Summary

- Consumer 1 has the right to generate as much as the externality she wants
- In the absence of any agreement, consumer 1 will generate h^*
- Consumer 2 will need to offer $T < 0$ to have $h < h^*$
- Consumer 1 will agree to have h iff: $\phi_1(h) - T \geq \phi_1(h^*)$
- Consumer 2 will choose her offer (h, T) to solve

$$\begin{aligned} & \underset{h \geq 0, T}{Max}\, \phi_2(h) + T \\ s.t\ \phi_1(h) - T & \geq \phi_1(h^*) \\ & \underset{h \geq 0}{Max}\ \phi_2(h) + \phi_1(h) - \phi_1(h^*) \\ FOC & : \phi_2'(h) + \phi_1'(h) = 0 \\ h^o & = \phi_1'(h) = -\phi_2'(h) \end{aligned}$$

- Consumer 1 pays $\phi_1(h) - \phi_1(0) > 0 \rightarrow$ to be allowed to set $h^o > 0$

- Consumer 1 receives $\phi_1(h^o) - \phi_1(h) < 0$ for setting $h^o < h^*$
- **Coase Theorem: If trade of the externality can occur then bargaining will lead to an efficient outcome no matter how PR are allocated.**

Assumptions

- Agents who suffers externalities are different than those who generates
- Generators of ext: Firms
- Experiencing ext: Consumers
- Partial equilibrium approach: Given price P of L tradable goods
- J firms generate the externality
- $\pi_j(h_j)$ derived profit function over the level of the externality
- I consumers, who have quasilinear utility function
- $\phi_i(\widetilde{h}_i)$ consumer i's utility over the amount of ext. $\widetilde{h}$
- Negative externality: $\phi_i'(\cdot) < 0, \phi_i''(\cdot) < 0, \pi_j''(\cdot) < 0$

- The externality experienced by each consumer is $\sum_j h_j$ (The total amount of the externality produced by the firm)
- At any CE, each firm will wish to set the h_j^* satisfying

$$\pi_j(h_j^*) \leq 0 \quad (\text{with equality if } h_j^* > 0)$$

- *In contrast, any PO allocation involves* $(h_1^o, \ldots, h_J^o)$

$$\underset{(h_1,\ldots,h_J)\geq 0}{Max} \sum_{i=1}^{I} \phi_i(\sum_j h_j) + \sum_{j=1}^{J} \pi_j(h_j)$$

$$FOC \quad : \quad \sum_{i=1}^{I} \phi_i'(\sum_j h_j^o) \leq -\pi_j'(h_j^o)$$

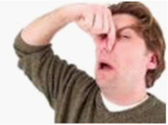
  - In the case of a Non-Depletable Externality, a market-based solution would require personalized markets for the externality, as in Lindhal eq. concept.
- In contrast, given adequate information, the government can achieve optimality using quotas or taxes

Presence of asymmetric information!

- Generators of ext: Firms
- Experiencing ext: Consumers
- $\phi(h, \eta)$ consumer's derived utility
- $\pi(h, \theta)$ derived profit function $\theta \in \mathbb{R}$
- η and θ are privately observed
- The ex-ante likelihoods (prob. distribution) of various values of η and θ are publicly known
- η and θ are independently distributed
- $\phi(h, \eta)$ and $\pi(h, \theta)$ are strictly concave in h for any given value of θ and η

Clarke's example: Sausage Company

Two types of individuals: affected (sensitive nose) and unaffected



Two types of firms: efficent and inefficient

Presence of asymmetric information

- Measurement of firm's benefits: $b(\theta) = \pi(h, \theta) - \pi(0, \theta) > 0$
- Measurement of consumer's cost from $\bar{h}$: $c(\eta) = \phi(0, \eta) - \phi(\bar{h}, \eta) > 0$
- $G(B)$ and $F(C)$ distribution functions of these two variables induced by the underlying probability distribution of η and θ
- density functions $g(b)$ and $f(c)$
- In the absence of an agreement $h = 0$
- Any arrangement that guarantees PO outcomes, the firm should allow to set $h = \bar{h}$ whenever $b > c$

Decentralized Bargaining

- h? when consumer cost is c

1. Firms will agree to pay T iff $b \geq T$
2. Consumers knows that if she demands a payment of T, the prob. that the firm accepts is equal the prob. that $b \geq T \rightarrow (1 - G(T))$

$$\begin{aligned} & \underset{T}{Max}(1 - G(T))(T - c) \\ Solution \quad : \quad & T_c^* > c \end{aligned}$$

- The aggregate surplus from ext: $\phi(h,\eta)+\pi(h,\theta)$
-

$$\begin{aligned} \text{Firm} &: \max_{h\geq 0}\pi(h,\theta) \\ st\ \ h &\leq \widehat{h} \\ Optimal\ \ Choice &: h^{o}(\widehat{h},\theta) \end{aligned}$$

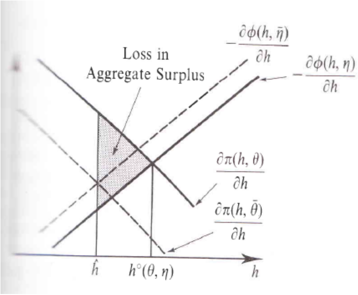
- The effect of the quota is to make h less sensitive to η and θ than is required by optimality. Firms will be insensitive to η

- The loss in aggregate surplus arising under the quota for types η and θ is given by:

$$\phi(h^q(\widehat{h},\theta),\eta)+\pi(h^q(\widehat{h},\theta),\theta)-\phi(h^o(\theta,\eta),\eta)-\pi(h^o(\theta,\eta),\theta)$$

$$=\int_{h^o(\theta,\eta)}^{h^q(\widehat{h},\theta)}\left(\frac{\partial\pi(h,\theta)}{\partial h},\frac{\partial\phi(h,\eta)}{\partial h}\right)dh$$

- The loss in aggregate surplus under a quota for types (θ,η)



Externalities

- $$\text{Firm:}\max_{h\geq 0}\pi(h,\theta) - t\times h, \quad Optimal\ \ Choice : h^t(t,\theta)$$

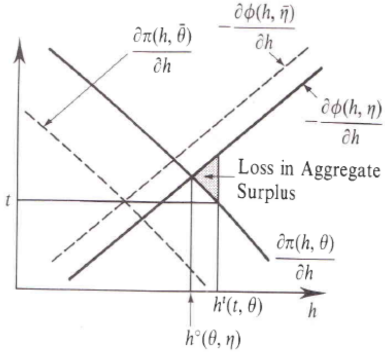
- The loss in aggregate surplus arising under the tax for types η and θ is given by:

$$\phi(h^t(t,\theta),\eta) + \pi(h^t(t,\theta)),\theta) - \phi(h^o(\theta,\eta),\eta) - \pi(h^o(\theta,\eta),\theta)$$
$$= \int_{h^o(\theta,\eta)}^{h^t(t,\theta)} \left(\frac{\partial\pi(h,\theta)}{\partial h}, \frac{\partial\phi(h,\eta)}{\partial h}\right) dh$$

- But now assuming that a tax is set a $t = -\frac{\partial\phi(h^o(\overline{\theta},\overline{\eta}),\overline{\eta})}{\partial h}$

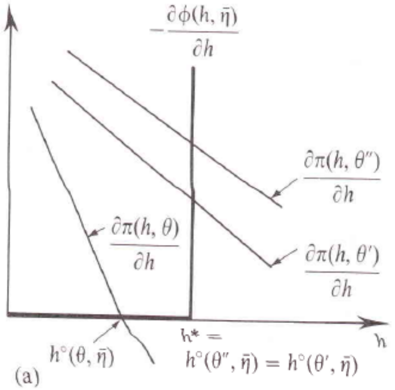
Externalities

- the loss in aggregate surplus under a tax for types (θ, η)



- Note that quotas and taxes, the level of externality is responsive to changes in Mg benefits but not to changes in the Mg cost of the consumer.

- Quota or Tax performs betters?
    - It depends!
- Quota $h = h^*$ Maximizes aggregate surplus for all θ



1. Tax $t = t^*$ maximizes aggregate surplus for all θ